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SALTS, SOIL-COLLOIDS, AND SOILS

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To reclaim permanently and manage alkali lands successfully, or to use fertilizer salts with intelligence and profit, requires a consideration of the effects of these salts on the physical condition of soils. Not only must we bear in mind the effects of the salts while present in the soil, but also the condition that may result when these same salts are removed partially or wholly by natural or artificial drainage. In our experience in California this latter phase has proved to be a more difficult and perplexing problem than the former. The importance of this subject justifies the publication of this brief discussion, prior to the appearance of the detailed statement with reference to three years of work which we have carried out upon it.

We know that, if a salt is added to a soil suspension, certain obvious effects are produced. Thus the common acids and their salts with the alkalies, the alkaline earths, and the heavy metals, as well as many other salts, flocculate suspensions in water of the soil mass. We associate this flocculated condition with an improvement in the working qualities of the soil. Another class of salts, those which give rise to an alkaline reaction, such as the alkaline hydrates, and as assumed, the alkali carbonates, possess within certain concentrations, the opposite power of deflocculating soil suspensions. Field soils so affected reflect this condition by their imperviousness to water and their inferior cultivating qualities.

Undoubtedly these phenomena are to be associated with the soil particles of such a fine state of division as to make their relatively large surface a factor of special importance. Possibly such particles may properly be designated colloids. Hence in attempting to explain

salt effects on soils, we must look to the laws formulated from studies on colloidal materials. Foremost among these may be mentioned those concerning the migration and electrical charges of colloids by Picton and Linde[1] and the subsequent contribution by Hardy,[2] setting forth the important rule that ions which carry charges opposite to those associated with particular colloids will act as precipitants for those colloids. This rule could be anticipated from the previous discovery of Schulze[3] that the flocculating power of an ion is in some measure dependent upon its valence. More recently Tolman[4] has gone over the field anew, and, by assuming that stable colloidal solutions exist only when the surface tension between the particles and the liquid is equal to zero, has developed a very interesting and highly plausible theoretical explanation for the phenomena occurring in dispersoid systems. By means of these facts we are able to predict with some assurance the effect accompanying the constant presence of a particular salt on the physical condition of a soil. It is of particular significance in this connection to remember, however, that these conceptions regarding the behavior of colloids are based on systems wherein the salt remains in contact with the particles. Such is rarely the case in normal field soils; for various agencies are constantly affecting the position of the salts, and as we shall see the leaching out of certain salts from a soil seems to produce an entirely new system, to which the laws referred to may not be strictly applicable. At least the conditions are sufficiently puzzling to merit the following discussion.

If a soil to which sodium chloride or sulphate has been added is subjected to leaching processes, it assumes an entirely new set of physical properties, characterized by a more or less complete deflocculation.[5] Evidently a new system, differing from that composed of the soil in contact with its own solution, or in contact with the salt solution, has resulted from the treatment. This phenomenon was first observed by us in a field experiment consisting of cylinders containing a clay loam soil at Davis, Cal., to which solutions of sodium chloride, sulphate, and carbonate had been added. It will suffice here to say that the salt-treated soils became very impervious to water during the winter season, and when dry were extremely intractable. The untreated control soils were easily managed at all times.

Results similar to those observed in the field were secured in the laboratory when sodium lactate, acetate, nitrate, chloride, sulphate, carbonate, and hydrate, potassium chloride, and sulphate, and ammonium sulphate were washed from the Davis soil. Magnesium chloride is also slightly effective in the same direction, while calcium chlo-

ride and barium chloride, when similarly leached from it, seem to leave the soil in its original condition. Not only were various salts tried, but also various soils with the same salt. Thus the washing out of sodium chloride from the Davis clay loam, the Berkeley adobe, the Anaheim sandy and the Oakley blow sand brought about a marked increase in the diffusion of the soil colloids. Apparently, then, we are dealing with a phenomenon of general application.

Turning to the literature we find reports of some interesting observations which concern the phenomenon under consideration, Adolph Mayer[6] noted a decrease in percolation when certain salts are washed from soil; and according to Van Bemmelen[7] the same thing occurs when clays or the hydrated oxides of tin, silica, and manganese are treated in the same manner. He also calls attention to the turbidity of the filtrate which appears as the salts are washed out. A similar turbidity has been observed by us in the filtrates from soils being washed free of salts. Warington[8] also refers in a general way to the appearance of somewhat similar phenomena when soils previously treated with acids are washed with water. Some further observations pertaining to this subject have also been recorded by Warington,[9] Hall,[10] Krüger,[11] McGeorge,[12] and others. Their observations dealt in the main with fertilizer salts and particularly with the after effects of sodium nitrate. In general, the ill effects of long continued use of this salt, are attributed to the formation of sodium carbonate.

Finding that the salt and water treatment of soils referred to brought about such striking results, some experiments were undertaken with the view of studying the mechanism by which these effects are produced. These investigations followed certain theories which had been advanced to account for the soil conditions noted above.

If a soil which has been treated as outlined above is suspended in water it yields a suspension much richer in solid matter than a suspension similarly formed from the untreated soil. Boiling the normal soil in distilled water produces a somewhat similar effect. Likewise, the same general condition of diffusion appears if NaOH in certain concentrations is added to a suspension of the normal soil. We also found that it required more NaCl to flocculate the suspension of a soil from which this salt had been previously leached than to flocculate a suspension of the normal soil. Evidently the colloids of the salt-treated soil are in a high state of diffusion. Furthermore, there is some indication that colloids of a new type, or perhaps additional colloids of the same general type, have been formed.

In either case an increase in the internal surface of the soil would

be expected. That such an increase had occurred seemed evident from the data secured by the use of Briggs'[13] centrifugal method for moisture-equivalents. However, a study of the hygroscopic coefficients of the variously treated soils failed to reveal any increase in surface. In all probability Mitscherlich's[14] method would be the most appropriate one for obtaining information regarding this point, but it has not yet been tried by us.

The interchange of ions when salt solutions are placed in contact with soils forms the working basis for the hypothesis that new colloids have been formed. The filtrates from soils which have received salt applications followed by leaching with water contain calcium and magnesium in amounts chemically equivalent[15] to the sodium taken up in exchange therefor by the silicates. It is to this new sodium silicate compound as the first possibility that we attribute the increased colloidal properties noted above. Potassium and ammonium salts seem to produce similar compounds, while calcium and barium salts under the same conditions do not produce the colloidal hydrated silicates. These new substances, together with the normal soil colloids, remain in a flocculated condition as long as free acid[16] or an excess of the soluble salts added or those formed by the interchange is present.

As flocculated colloids their effect on the physical condition of the soil is either beneficial or passes unobserved. Hence as long as salts are present in sufficient quantity no objectionable feature appears, but when these flocculating agents are removed by leaching with water, the colloids become diffused and offer great resistance to the further passage of water.

In explanation thereof, we might assume in accordance with Tolman's ideas that the normal lyophobic soil colloids when suspended in water have a positive surface-tension, hence greater than zero, but that the new hydrated sodium silicate when suspended in the same medium automatically provides by hydrolysis a sufficient quantity of OH ions to lower this surface-tension and to bring it nearer the zero value, so that the colloids, in this case, would have a far greater tendency to become and to remain dispersed. This assumption requires the consideration that both the normal colloids and those resulting from the interchange of ions carry negative charges, which brings us immediately to our next hypothesis which deals in the main, with the nature of the suspending medium, rather than with the changes occurring in the solid substances.

This hypothesis is largely based on the contention that the behavior of soils under the influence of salts agrees in some measure with the laws which are thought to govern the behavior of dispersed systems to

which salts have been added. Obviously this would be the case when we are considering suspensions of the soil mass in various solutions. Some doubt, however, may be expressed as to the validity of this statement when a soil of normal moisture content is compared to the dispersoid of a disperse system, or to a soil suspended in a liquid. For in the normal soil such factors as the surface tension of the liquid-air surface may be of such magnitudes as to materially alter conditions so that the laws referred to would not be applicable to such systems. In order to see how far the analogy can be drawn let us consider, for example, clay. Clay has been shown to be a lyophobic colloid which assumes the negative charge of OH ions when suspended in water. The addition of more OH ions as by NaOH within certain limitations lowers the solid-liquid surface-tension so that a greater diffusion results. If a positive ion is added to the system, the OH ions are neutralized, the surface-tension increases, and the clay particles increase in size until they finally settle to the bottom of the vessel. So far the well known facts agree with the theory and, moreover, it is evident that the nature of the medium of suspension is an important factor. When a salt, such as sodium chloride, is added to a soil and subsequently leached out, it may be possible that the medium of suspension has been materially altered. The selective adsorption of the sodium by the soil may eventually give rise to OH ions, in which case the soil would become more or less diffused. By means of the hydrogen electrode we have been able to obtain specific data with respect to the quantity of OH ions present in the medium of suspension. Our results so far indicate that there is not a sufficient quantity of OH ions in the solution bathing the soil particles of the salt treated soil, to account for its unusual degree of diffusion. These data bear significantly on the general behavior of the soil suspension under various conditions.

If a soil which has been previously treated with salt and leached with water is subjected to drying at 130° C., it loses only a small portion of its diffusible colloidal material. Whatever other bearing this may have upon the subject, it at least indicates that the change is of a permanent character.

In the course of these investigations we have also noted that salt solutions of low concentration (less than $\frac{1}{1000}$ normal) have little or no effect on a soil suspension, so that the mere dilution of the salt solutions can not account for the effects observed. Moreover, it has been found that it is not necessary to wash the soil entirely free from salt in order to produce the deflocculated condition.

One other interesting observation with respect to the difference in behavior of sodium hydroxide and carbonate came to our notice. Sodium

hydrate is a stabilizing agent in a fairly wide range of concentrations. Sodium carbonate, on the other hand, failed to stabilize at any concentration. Both substances are precipitating agents at concentrations greater than $\frac{1}{15}$ normal. Our studies with the hydrogen electrode include systems of soil suspended in various concentrations of these substances. In case sodium hydroxide or carbonate is added to a soil the bases combine directly with the silicates forming addition-compounds, as has been also pointed out by Sullivan (loc. cit.[15]). Water or carbonic acid is the end-product, hence the washing process so essential for the appearance of diffusion in soils after the use of neutral salts is not so necessary in the case of alkaline salts.

Briefly stated the experiments upon which the above summary is based have enabled us to throw upon the subject of salts in relation with soil colloids a light which has never before, particularly by soil chemists, been cast upon it. Not only is the way opened for an extensive experimental field in the physical chemistry of the soil, but there are indications that the principles involved, part of which are for the first time gathered together and correlated by us, will be of profound practical significance in the important subjects both of 'alkali' and of fertilizer salt applications to soils. Many of the perplexing aspects of the physical effects of continued nitrate of soda and other fertilizer applications to soils, as well as those following the leaching out of alkali, will, we hope, be largely cleared up by the investigations soon to be reported at length.

My thanks are due to Dr. Chas. B. Lipman for many helpful suggestions and for his critical reading of the manuscript.

[1] *London, J. Chem. Soc.*, **71**, 568 (1897).

[2] *London, Proc. R. Soc.*, **66**, 115 (1899).

[3] *J. prakt. Chem., Leipzig*, Ser. 2, **25**, 445 (1882).

[4] *J. Amer. Chem. Soc.*, **35**, No. 4 (1913).

[5] Free has already discussed flocculation as a matter of degree—*Philadelphia, J. Frank. Inst.*, 1910.

[6] *Forsch. auf dem Gebiete der Agrik. Physik*, Vol. 2, 1879, p. 251.

[7] *J. prakt. Chem.*, Ser. 2, **23**, 388 (1881).

[8] *Physical Properties of Soil*, Cambridge Univ. Press, 1900, p. 30.

[9] Loc. cit.

[10] *The Soil*, 1910, p. 252; also *London, J. Chem. Soc.*, **85**, 964 (1904).

[11] Cited from *U. S. Dept. Agric. Off. Exp. Sta. Rec.*, **29**, No. 8, 719 (1908).

[12] *Hawaiian Agric. Exp. Sta., Bull.* No. 35, 1914.

[13] *U. S. Dept. Agric. Bur. Soils, Bull.* No. 45.

[14] *Bodenkunde für Land-und Forstwirte*, p. 51.

[15] After reviewing this subject Sullivan draws the following conclusion—"So far as the evidence goes, then, the action of silicates, clay, and other constituents of the earth's crust on solutions of such salts as do not dissolve in water with alkaline reaction consists in an equivalent exchange of bases." *U. S. Geol. Sur., Bull.* No. 312, p. 27 (1907).

[16] For a discussion of the presence of free acid in salt solutions in contact with soils the reader is referred to the work of Parker, *J. Agric. Res.*, **1**, 179 (1913).

THE CHILD AND THE TRIBE

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In a previous article, 'The Indian and Nature,' (These PROCEEDINGS, 1, 467) I outlined the Siouan Indian's conception of Nature and his attempt to reflect that conception in his tribal rites and gentile organization. The new born child was regarded as 'a new life' that had come 'into the midst' of the various forms of nature. The babe had no personal place in the tribe until it became old enough to pass through the ceremony called by the Omaha 'Turning the Child,' by which it entered the tribe and received one of the names belonging to its father's village (gens). This change in the status of the child was effected through the presence during the ceremony of those dual forces by which Wakonda, it was believed, had brought all forms into being and made their perpetuity possible, namely, the masculine element resident in the Sky, and the feminine element dwelling in the Earth, the former was represented by the 'Four Winds' invoked by the priest in the opening ritual song, and the latter was present in the Stone upon which the child was 'turned' by the priest and made to face successively the four directions.

It is to be noted, that during this important ceremony the parents of the child were not present within the sacred tent, nor did they have any part in the rite. The dual forces alone became responsible for the birth of the child into the tribal organization. No stronger or more practical evidence could be given to show the vital character of the Indian's belief in what he conceived to be the established order of Nature, than this surrender of his child to that order as manifested in the rites and organization of the tribe.

Through this ceremony, not only did the child enter into its place in the tribe, but at that time it took its first authorized steps in 'the path of life.' The Omaha ritual song that accompanies these intial steps of the child, sets forth wherein, the Indian believed, lay the power through which man received the ability 'to stand' and to 'go forth' in life. (A free translation of the song is here given.)

Here unto you has been spoken the truth,
Because of this truth, you shall stand.
Here, declared is the truth!
Here in this place has been shown you the truth;
Therefore, Arise, go forth in its strength.

Studying this rite in connection with the one used to introduce the child to the cosmos, given in the previous article, together with the various tribal ceremonies of the Siouan group of tribes, it becomes clear that 'the truth' referred to in the ritual song, given above, is that man is an integral part of an order established and sustained by the invisible, undying power, called Wakonda. When the child, through this ceremony, entered the tribe it became a recognized member of the cosmic family as established by Wakonda and drew its strength to pursue the path of life from this truth. Henceforth help could be asked of Wakonda and be brought to the growing child through the performance of the duties laid upon it as a member of the tribe. The children thereafter grew up under the influence of the Indian's belief in an ever present though invisible Wakonda.

For the male child, there was a supplementary rite which followed that through which he was born into the tribe. This ceremony pertained to the life of the boy as a future warrior and protector of the tribe. The rite was called, by the Omaha, 'Wébashna,' to cut the hair. There is evidence indicating that this rite belongs to a later period in the evolution of the tribe than the preceding ceremony of 'Turning the Child.' The limitation of space forbids the presentation of evidence showing how, through the specializing of duties with their attendant ceremonies, the political growth of the tribe slowly developed. In the following rite we discern that the warrior has been raised above the mere fighter, he has become the man of power, one entitled to be placed in close relation to those natural manifestations of power seen in the fury of the storm and heard in the rolling of the thunder. In the efforts toward political unification of the tribe, such rites as those connected with Thunder would conduce to the welding of the people, by the inculcation of a common dependence upon a powerful god and the putting of a sign of consecration to this god upon the head of every male member of the tribe.

In the ceremony of Wébashna, the priest took the boy to the space within the sacred tent west of the fire, there, both facing the east, the priest gathered a tuft from the crown of the boy's head, cut it off and laid it away in a parflesh case, which was kept as a sacred repository, singing as he cut the hair the following ritual song, explanatory of the act. (A free translation is given.)

Grandfather! far above, on high,
The hair, like a shadow, passes before you;
Grandfather! far above on high.
Dark like a shadow, the hair sweeps before you into the midst of your realm.

Grandfather! far above, on high,
Dark like a shadow, the hair passes before you,
Grandfather! dwelling afar, on high,
Like a dark shadow, the hair sweeps before you into the midst of your realm.
Grandfather! far above, on high,
The hair, like a shadow, passes before you.

The hair laid away in the sacred case was sent symbolically to the Thunder dwelling 'far above, on high,' who was addressed as 'Grandfather,' a term of highest respect. The severing of the lock, implied the consecration of the life of the boy to Thunder a symbol of Wakonda the power that controlled the life and death of the warrior. Every man had to be a warrior and thus become a part of the only bulwark of defence to the home and to the tribe. Because of the belief in the continuity of life, a part could stand for the whole, therefore in this rite, by the cutting of a lock of the boy's hair and giving it to Thunder, the life of the child was ceremonially given into the keeping of Wakonda.

The hair of a person was popularly believed to have a vital connection with the life of the body, so that any one becoming possessed of a lock of hair might work his will on the individual from whom it came. In the light of customs that obtained among the people, the hair, under certain circumstances, might be said to typify life.

It is to be noted in this connection, that later, when the hair was allowed to grow on the lad's head, a lock on the crown was parted in a circle from the rest of the hair, kept constantly distinct and always neatly braided, no matter how frowzy and tangled the remainder of his hair might be. It was upon this lock that the war honors of the warrior were worn. It was this lock that was cut from the head of a slain enemy and formed the central object in the triumph ceremonies, for the reason, that it preeminently represented the life of the man who had been killed in battle.

The ritual song that followed the cutting of the lock of hair from the head of the boy makes clear the meaning of the act. (The following is a free translation.)

What time I will, then only then,
A man lies dead, a gruesome thing;
What time I will, then only then,
A man lies dead, a gruesome thing;
What time I will, then only then,
Like a shadow dark, the man shall lie;

What time I will, then suddenly,
A man lies dead, a gruesome thing;
What time I will, then only then,
Reddened and stark, a man lies dead;
What time I will, then only then,
A man lies dead, a gruesome thing.

The word meaning 'dark like a shadow' used in the preceding song to indicate the lock of hair cut from the boy's head as a symbol of his life and offered to Wakonda, is in this song applied to the man, who 'like a shadow dark' 'shall lie' when his life has been taken by the god. The use of the word bears out the meaning of the act that accompanied the singing of the previous song, that, by giving the lock of hair, the life of the giver was placed in the keeping of the Thunder as representing Wakonda. This song shows that the god intends to do as he wills with the life that has been offered him. There are other ceremonial songs which iterate the belief that a man dies only when Wakonda so decrees.

These two rites which connected the life of the child with that of the tribe also initiated the indirect religious teaching as given to Indian children. Of course the little ones who passed through these rites were too young to understand their meaning but the vague memory of the dramatic acts was augmented as year by year the ceremonies were repeated. The cover of the sacred tent where these rites took place was always turned up from the ground, thus exposing the frame work of poles, through which all that took place within the tent was easily seen and the songs heard by the crowd of old and young people that gathered to witness the ceremonies.

In a like indirect manner the children acquired familiarity with the composite character of the tribal organization, which was primarily religious in character and to which the political system was subsidiary. Mention was made in the previous article, already referred to, of the custom of the father cutting his son's hair in an established manner, one that was meant to typify the sacred symbol of his village (gens). This custom of so cutting a boy's hair was kept up from the time the child passed through the rites that made him a member of the tribe, until the period of his second dentition. As the children played together, because of the queerly cropped head of their companions, they came to know to which village (gens) each playmate belonged. They also became accustomed to the use of symbols and to know what the symbolic cut of a boy's hair represented, whether it indicated an animal, a bird,

or a cosmic force, as the Wind. Thus unconsciously, the children drifted into the atmosphere of the Indian's belief, that he, as man, was included in the great unity of Nature's family.

The children also learned the place assigned to each village (gens) in the tribal order. While the people were on the great tribal buffalo hunt and during the elaborate ceremonies that took place at its close, the tribe camped in the ceremonial order. At this time the Omaha encampment was in a circular form with an opening toward the east. The north half of the circle, with its five villages (gentes), formed the Sky division; the south half of the circle, with its five villages (gentes), formed the Earth division. In this way, the dual forces, that were regarded as formative instruments in Nature, were represented. There is an interesting bit of evidence as to the use of native logic observable in the following arrangement whereby all the leaders in the ceremonies connected with the material welfare of the people, as, the securing of food by cultivating the maize, hunting, etc., and those, relating to the secular government and to war, were men who belonged to the Earth division; while those rites that were concerned with the securing of help from the supernatural and the maintenance of the tribal order, such as those cited above that pertain to the child, were in charge of leaders who belonged to the Sky division.

The vital existence of the abstract dual forces represented in the tribal organization was made real to the boys and fixed in their minds by the following custom. Fighting was ordinarily not encouraged among the lads, but there was one occasion when it was not interfered with by the elders, and that was when boys crossed the invisible line that divided the Earth from the Sky division while the tribe was camped in the ceremonial order. If a boy should be sent by his father on an errand that required the lad to cross this line, he did not dare to go alone, he would gather his friends, belonging to his own division, to go with him and help him in the battling that was sure to take place. It may truly be said that it was by hard knocks that this fundamental division of the tribe was beaten into the heads of the boys.

The children learned the rudiments of tribal beliefs through object lessons, such as those described above, rather than through oral teaching. A child was always obliged to observe certain forms of respect toward the symbol sacred to its village (gens), it must not touch or taste that which was associated with the symbol, for the symbol belonged to the tribal rites through which the people appealed to Wakonda, therefore, whatever was connected with the symbol was set apart from any familiar use. Beyond the insistence upon this form of respect to

the symbol sacred to his village (gens), there was no attempt to coerce the child to observe any rite or to direct its mind to think upon tribal beliefs.

A child gifted with an inquiring mind would be apt to ask questions concerning the symbols and the customs attending them of his father or mother and if his parents belonged to the thinking class, they might put the child's thoughts on the trail that in time would lead him to a knowledge and an understanding of the religious beliefs expressed in the tribal rites, otherwise, the boy or girl would grow up in unquestioning ignorance of the truths hidden within the dramatic rites of the tribe.

The sacred legend of the Omaha lays stress upon 'thought.' 'And the people thought,' was always the prelude to any change for the betterment of the tribe. The Indian thinker became the Indian seer, but, the man must seek for himself the path to that height.

THE CORRELATION OF POTASSIUM AND MAGNESIUM, SODIUM AND IRON, IN IGNEOUS ROCKS

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In its modern development the science of petrology is devoted, in great part, to the physico-chemical study of igneous rocks, regarded as congealed solutions, and of the conditions of separation of their constituent minerals. Distinct from, and in some respects more general than, this, is the study of the distribution of the chemical elements of which they are composed.

This distribution may be spatial; that is, according to the occurrence of groups of rocks, or rock series, over different areas of the earth, each with its dominant chemical characters. Such districts are known as petrographic provinces or comagmatic regions. Their study involves the consideration of the original homogeneity or heterogeneity of the globe, as well as, possibly, certain tectonic factors, as suggested by Harker, Becke, and others, and does not concern us here.

Or the distribution may be according to a chemical correlation, expressed by the presence of certain pairs or groups of elements, which vary together, however they may be spatially distributed. In other words, in rock magmas, solidified as igneous rocks, as well as in their constituent minerals, certain of the elements are found to be generally and persistently associated together, the one being abundantly present

when the amount of its correlate is high, and the converse, but not usually so when the rock magma is dominantly characterized by the abundance of another, non-correlated element, in which case another element, correlate with it, will appear as the co-dominant one.

This phase of petrology, which is of a complexity probably much greater than is superficially apparent both as to character and cause, has, as yet, received comparatively little attention. J. H. L. Vogt,[1] J. F. Kemp,[2] L. de Launay,[3] and W. F. Hillebrand[4] have discussed the subject, but have confined their observations almost wholly to the distribution of the elements according to the relative amount of silica (silicity) shown by the igneous rocks.

A few years ago I showed[5] that these relations are far more complex than the simple one based on the silicity. Thus it is clearly obvious from a study of rocks and rock analyses that, among other relations, lithium, zirconium, the elements of the rare earths, glucinum and others, are most abundant in dominantly sodic magmas, irrespective of their silicity; barium apparently in those dominantly potassic; titanium, vanadium, nickel and manganese in those high in iron; and chromium and platinum in those high in magnesium. It was also stated that "soda not uncommonly tends to vary with the iron oxides, while potash shows similar relations to magnesia."

The recent compilation of a second edition of *A Collection of the Chemical Analyses of Igneous Rocks*,[6] embracing all those published between 1883 and 1914 (numbering nearly 10,000), and soon to be published, has added greatly to the data available. This cumulative evidence now greatly strengthens, if it does not quite establish, the view that in igneous magmas potassium and magnesium on the one hand, and sodium and iron on the other, tend to vary together.

The scope of this paper does not permit the presentation in full or in detail of the data in regard to this or other correlations which have been noted, but a short statement may be of interest, in advance of a more extended publication, as calling attention to a phase of chemical petrology which has been somewhat neglected, but which would seem to be of fundamental importance.

The evidence is of two kinds; mineralogical and petrographical.

Among the important mineral groups, found in igneous rocks, which contain the elements in question, the law enunciated above holds good almost without exception. The sodic pyroxenes and amphiboles, such as ægirite, acmite, riebeckite, arfvedsonite, barkevikite, kaersutite, and basaltic hornblende, all show, along with practical absence of potassium, a strong preponderance of iron over magnesium, the latter usually

being present in these only in traces or at least in very small amounts. The few reliable analyses of augites from highly potassic rocks, on the other hand, show little soda, but a marked preponderance of magnesium over iron. It may be noted, incidentally, that there are no natural potassic pyroxenes or amphiboles corresponding to the sodic ones.

On the other hand in the ferromagnesian micas, biotite and phlogopite, the alkali metal is entirely or almost entirely potassium, the amount of sodium being generally negligible. In these there is usually a very marked predominance of magnesium over iron. Only in the potash mica, lepidomelane, which is characterized by high ferric iron, does iron dominate over magnesium, but it is to be noted that this variety seems to be confined to the sodic rocks. It is also noteworthy that there are no true soda micas, except paragonite, which is only known in metamorphic rocks. It may be added that the potassium-lithium mica, zinnwaldite or cryophyllite, carries much more iron than magnesium, and attention was called above to the fact that lithium is usually associated with sodic rocks.

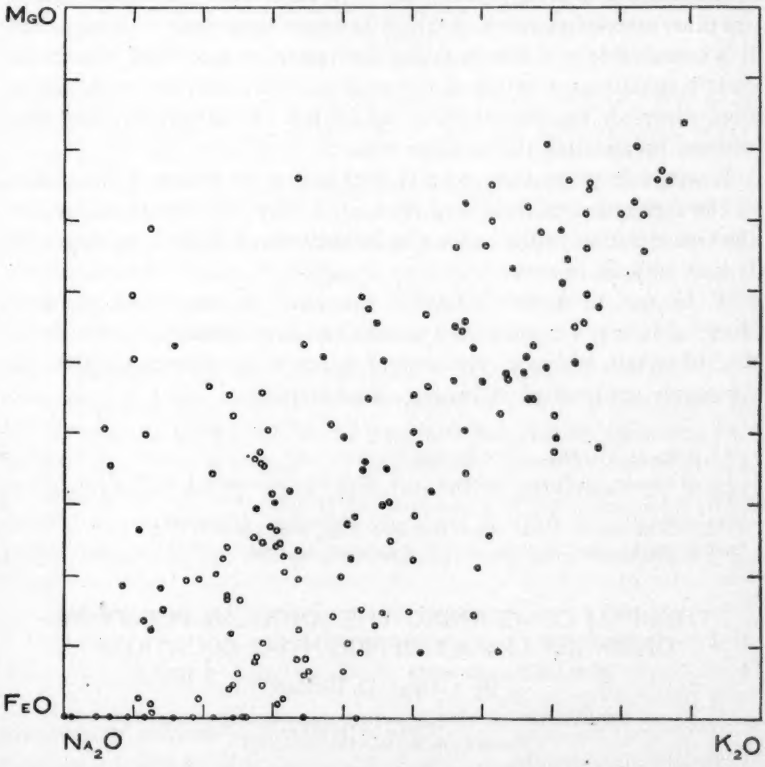
Of the multitudinous examples of the general law furnished by igneous rocks only a few prominent ones can be given here.

Highly potassic rocks are very rare, but the few known regions are characterized by high magnesium relative to iron. Such are the wyomingite, orendite and madupite of the Leucite Hills, Wyoming; the jumillite and verite of Murcia, Spain; the leucitic rocks of Celebes and of the Gaussberg, in the Antarctic; many of the leucitic rocks of the volcanoes of Western Italy (through here the dominance of magnesium over iron is not so pronounced). There are also many other scattering examples of syenites, shonkinites, minettes, leucitites, leucite tephrites, and other dopotassic rocks. Indeed the same general relation holds good even in such rocks as peridotites, pyroxenites and hornblendites, in which the amounts of magnesium and iron are very high and those of the alkalies very low.

Of the converse case, high sodium with high iron relative to magnesium, very striking examples are the highly sodic rocks of the Julianehaab District in Southern Greenland, in which magnesium is usually present only in traces and often wholly absent; the ijolites and other nephelite rocks of the Kola Peninsula in Finland; the nephelite syenites and related rocks of Ontario, Norway, the Transvaal, Portugal, Brazil, and other regions; and the sodic lavas of Pantelleria, Madagascar, and the Great Rift Valley in East Africa. Indeed illustrations of the law are so abundant among the highly sodic rocks as to make selection difficult.

In the figure is given the result of plotting nearly two hundred superior

analyses of rocks from all over the globe. To avoid complications due to the presence of considerable lime, a subject for further investigation, only peralkalic and some domalkalic rocks were selected, chiefly among the perpotassic, dopotassic, and persodic subrangs. The analyses belonging to the sodipotassic and dosodic subrangs are so numerous that only a very few of the many available were used. These analyses represent rocks from Greenland, Canada, Montana, Colorado, Wyoming,



Scotland, Norway, Finland, Bohemia, Italy, Madagascar, Celebes, Africa, Australia, Antartica and other localities, so that, as a whole, variations due to regional peculiarities are eliminated.

It may be added that the silica percentages of the rocks represented by these analyses vary from 39.24 to 73.68, thus covering nearly the entire range.

The clustering in a narrow band, extending diagonally from high soda and iron in the lower left quadrant, to high potash and magnesia

in the upper right, with the practical absence of analyses in the upper left and lower right quadrants, is very evident.

The chemical composition of igneous rocks is so complex and the number of their chemical constituents is so great; the conditions of differentiation and solidification are so varied and so complicated; and the knowledge of the application of the laws of physical chemistry to them is as yet so meager by reason of the lack of data, that some irregularities and apparent exceptions are to be expected. Furthermore there are other correlative relations which have not been dealt with here, and it is conceivable or indeed probable that one or more of these may under certain conditions, whether of composition, differentiation or solidification, supersede the one which is the subject of the present discussion without invalidating the truth of this.

It would be premature to enter here into a discussion of the causes of the correlation of these four elements. Any such would necessitate the consideration of the other correlations which have been detected. It may be said, however, that even though such concomitant variations may be due to similar solubility relations, or some such physico-chemical factor, yet that such similarities are themselves presumably due to certain intimate relationships between the elements which are commonly and somewhat loosely called affinities.

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THEOREM CONCERNING THE SINGULAR POINTS OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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Presented to the Academy, October 15, 1915

In earlier papers I have considered the effect of a linear transformation of dependent variables upon the solutions of ordinary linear differential equations in the vicinity of a singular point.

This type of transformation led me to the notion of *equivalence* which is fundamentally important for the classification of singular points.

Ordinary linear differential equations also preserve their form under an arbitrary transformation of the independent variable. I shall prove here that this second type has no additional significance for the purposes

of classification since the solutions are carried over into the solutions of *equivalent* differential equations by such transformations.[1] The precise statement of the result is contained in the following

THEOREM. *Let $Y(x)$ be a matrix[2] of functions $y_{ij}(x)$ $(i, j = 1, 2, \cdots, n)$, each analytic for $|x| \geqq r$, with determinant $Y(x)$ not zero for $|x| \geqq r$. A necessary and sufficient condition that $Y(x)$ forms the matrix solution of a linear differential system*

$$\frac{dY}{dx} = A(x)\, Y,$$

where the elements $a_{ij}(x)$ of $A(x)$ are analytic $(q = 0)$ or have a pole of at most the qth order $(q > 0)$ at $x = \infty$, is that, for every function $\varphi(x)$ of the form

$$\varphi(x) = x + l\, x^{-q} + m\, x^{-q-1} + \cdots,$$

the matrix $M(x)$ defined by the matrix equation

$$Y(\varphi(x)) = M(x)\, Y(x)$$

is composed of elements $m_{ij}(x)$ analytic at $x = \infty$.

Let us prove first that the condition is *necessary*.

The function $M(x)$ defined for any choice of $\varphi(x)$ is single-valued as well as analytic for $|x|$ sufficiently large. In fact, when x makes a circuit of $x = \infty$, $Y(x)$ is altered to $Y(x)C$ where C is a matrix of constants of determinant not zero. But since $\varphi(x) - x$ remains finite as x becomes infinite, $\varphi(x)$ will make essentially the same circuit if $|x|$ be sufficiently great. Thus $Y(\varphi(x))$ changes to $Y(\varphi(x))C$, and $M(x)$ is unaltered in value.

Now the expression for $M(x)$

$$Y(x_0)\, Y^{-1}(x_m) \qquad (x_0 = \varphi(x);\ x_m = x)$$

may also be written as a matrix product

$$[Y(x_0)\, Y^{-1}(x_1)]\, [Y(x_1)\, Y^{-1}(x_2)] \cdots [Y(x_{m-1})\, Y^{-1}(x_m)].$$

Here we will assume that the values $x_0, x_1, x_2, \ldots, x_m$ of x are equally spaced, each after the first differing from its predecessor by $\Delta x = (x_1 - x_m)/m$.

From the differential equation itself, it is clear that

$$Y(x_i) - Y(x_{i+1}) = \Delta x[A(x_i) + E_i]\, Y(x_{i+1}) \quad (i = 0 \text{ zero}, 1, \cdots, m-1)$$

where E_i will have arbitrarily small elements if Δx is small enough.

For $|x|$ taken large enough, it is also clear that we will have an equality of the form

$$|a_{ij}(x)| < a|x^q| \qquad (i,j,=1,2,\cdots,n)$$

for the elements of $A(x)$. Hence if I denotes the matrix with elements $\delta_{ij}(\delta_{ij}=0,\ i\neq j;\ \delta_{ii}=1)$, and if A denotes the matrix whose elements are all equal to the positive constant a, we find that the elements of the matrix

$$Y(x_i)\,Y^{-1}(x_{i+1}) \quad \text{or} \quad I+\Delta x(A(x_i)+E_i)$$

are less in absolute value than the corresponding elements of the matrix

$$I+2|\Delta x|A|x_i|^q.$$

Also on account of the form of $\varphi(x)$ we have for $|x|$ large enough

$$|x_0-x_m| \leqq k|x_i|^{-q},\ (k \text{ independent of } i \text{ or } x).$$

and thence

$$|\Delta x| \leqq \frac{k}{m}|x_i|^{-q}.$$

Substituting this value of Δx above, we conclude that the elements of $Y(x_i)Y^{-1}(x_{i+1})$ are less in absolute value than the corresponding elements of

$$I+\frac{2k}{m}A.$$

Thus, by the product formula, $M(x)$ will have elements whose absolute value is less than that of the corresponding elements of

$$\left(I+\frac{2k}{m}A\right)^m$$

whose elements are obviously limited in absolute value for all values of m.

Consequently the elements of $M(x)$ are limited in absolute value for $|x|$ large enough, and are necessarily analytic at infinity.

Let us show now that the stated condition is *sufficient*.

To this end we will consider the family of functions

$$\varphi(x)=x+\rho x^{-q}$$

where ρ is a parameter.

Defining $M(x,\rho)$ by the equation

$$Y(x+\rho x^{-q})=M(x,\rho)\,Y(x),$$

we observe that $M(x,\rho)$ is made up of elements single-valued and analytic in x and ρ if

$$|x| \geqq {}^1 + r, \qquad |\rho| \leqq 1,$$

inasmuch as in this event

$$|x + \rho x^{-q}| \geqq |x| - |x^{-q}| \geqq |x| - 1 \geqq r$$

Moreover, in view of the stated condition, the elements $m_{ij}(x,\rho)$ of $M(x,\rho)$ are analytic in x at $x = \infty$.

Consider now the integral formula

$$\frac{1}{x} m_{ij}(x,\rho) = \frac{-1}{2\pi\sqrt{-1}} \int \frac{m_{ij}(\xi,\rho)\, d\xi}{\xi(\xi - x)} \qquad (|x| > 1 + r)$$

where the integration is performed around the fixed circle C: $|\xi| = 1 + r$ in a positive sense. This reduces to the Cauchy integral formula by the substitution $x = 1/x'$, $\xi = 1/\xi'$, which holds since $m(x,\rho)$ is analytic for $x \geqq 1 + r$ and at $x = \infty$.

We now perceive from the form of the integrand that $m_{ij}(x,\rho)$ must be analytic in x *and* ρ for $x = \infty$.

Differentiating the equation of definition for $M(x,\rho)$ with respect to ρ we obtain

$$x^{-q} \frac{d}{dx} Y(x + \rho x^{-q}) = \frac{d}{d\rho} M(x,\rho)\, Y(x).$$

The elements of $dM(x,\rho)/d\rho$ are also analytic in x and ρ at $x = \infty$. Letting ρ approach zero we obtain, for $|x| > 1 + r$,

$$x^{-q} \frac{dY(x)}{dx} = \frac{d}{d\rho} M(x,0) Y(x)$$

which establishes at once that $Y(x)$ is the solution of a differential system of the desired form.

[1] This result was partly known to me in 1908. For a special case, see *Trans. Amer. Math. Soc.*, 14 462–476 (1913), in particular pp. 475–476.

[2] For the elements of the theory of matrices here used see Schlesinger, *Vorlesungen über linearen Differentialgleichungen*, pp. 18–19.

A QUANTITATIVE STUDY OF CUTANEOUS ANALGESIA PRODUCED BY VARIOUS OPIUM ALKALOIDS

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Presented to the Academy, October 20, 1915

The pharmacological and physiological literature contains but few quantitative and scientifically accurate comparisons of the narcotic or analgesic properties of the various opium alkaloids. This is chiefly due to a lack of an adequate method for studying the subject. The observations so far recorded are of a clinical character, and the conclusions drawn, even in the cases where alkaloids of reliable purity were used, show the greatest diversity of opinion. Thus, Claude Bernard[1] regarded narcein, an inert substance according to the majority of investigators, as a powerful narcotic; again Fronmüller[2] ranked narcotin in activity as next to morphin; while Baxt[3] extolled the wonderful pain-relieving virtues of papaverin.

In the present investigations we have made use of a large Baltzer inductorium for producing quickly and conveniently finely graded pain stimuli. Having standardized our apparatus with the help of Dr. C. W. Hewlett of the Physical Laboratory of this University, we were able to express the values of these stimuli, quantitatively, in Henrys or C. G. S. units, and in this way compare the strengths of the stimuli re quired to produce pain under various conditions.

The first step in the investigation was to determine whether a sufficiently constant pain threshold could be established. Martin[4] and his coworkers have studied the threshold of electrocutaneous sensation with the induction current, by dipping a finger in a liquid electrode, and have found that a definite sensory threshold can be established, subject to physiological diurnal, nocturnal, and fatigue variations. Furthermore, Martin, Grace and McGuire[5] in the only pharmacological study by this method, have found a definite lowering of the sensory threshold after administration of acetphenetidin by mouth.

In our work we have made use of fine platinum electrodes, studying the effect of the induced current on individual pain points or groups of pain points in four different regions of the body. In this way our chances of error were lowered fourfold. The points most convenient for study employed by us were the skin on the back of the hand between thumb and forefinger, the tip of the nose, the tip of the tongue, and the lips.

In complete agreement with the localization of pain points on the surface of the body, as described by v. Frey,[6] and others, we have found that at any given spot of the body a definite pain sensation can be elicited by changes in mutual inductance of sufficient intensity, and that for every group of pain points a pain sensation of exactly the same quality and intensity can be elicited by exactly the same intensity of electrical stimulus. By practice we were able to distinguish changes in pain sensation produced by moving the secondary towards or away from the primary coil a distance of not more than 0.1 cm.

Through numerous observations it was established that the normal pain threshold remains surprisingly constant for many hours in succession, and, through a series of other experiments, lasting over 25 hours each, the diurnal and nocturnal variations were found to be very slight.

Having determined the normal pain threshold in any given experiment, a drug was administered by subcutaneous or intramuscular injection, and, after its absorption, repeated readings were made. In this way a rise or fall in the pain threshold, or an analgesic or hyperalgesic effect respectively, was detected and measured. The experiments were performed on Dr. Macht and two medical students, Messrs. N. B. Herman and C. S. Levy.

Being fully aware of the fact that certain subjective elements, inherent in the character of the investigation, enter into our experiments, we have taken all possible precautions to eliminate errors arising from this source, and have made numerous control experiments.

Each experiment was carried out in the same room, and under perfectly constant conditions. Readings were always taken with the subject in the same position, and the subject was never allowed to look at the apparatus, but sat either with eyes closed or fixed on some distant point. The electrodes employed were of course the same in any one experiment; the distance, between the electrodes was kept fixed; the pressure with which they were applied to the surfaces was kept constant; the direction of their application was the same; and the wetness of the surfaces stimulated was maintained the same as nearly as could be judged.

When a drug was administered, the subject was ignorant of its nature. Furthermore, as controls, normal saline and other inactive substances were often substituted in place of the drug without the subject's knowledge. It may be remarked in passing, that owing to the conflicting experiences of previous observers, we could not know the true pharmacological action of most of the alkaloids studied; thus further eliminating any subjective bias.

Action of Opium Alkaloids Individually.—By the above methods the six principal opium alkaloids, morphin, papaverin, codein, narcotin, narcein and thebain were studied. Administered in moderate therapeutic doses, it was found that in respect to their analgesic power beginning with the strongest and ending with the weakest they ranged themselves in the following order: Morphin (10 mgs.)→Papaverin (40 mgs.)→Codein (20 mgs.)→Narcotin (30 mgs.)→Narcein (10 mgs.)→ Thebain (10 mgs.).

In respect to morphin in one of us an idiocyncrasy was noted. The subject (N. B. H.) was rendered hypersensitive to pain, and this effect could be measured quantitatively. This was an interesting confirma-

TABLE SHOWING THE MAXIMAL EFFECTS OF THE VARIOUS DRUGS USED FOR D. I. M.

TIME OF READING	MORPHIN SULPHATE 10 MGS.	MORPHIN SULPHATE 5 MGS.	PAPAV. SULPH. 40 MGS.	CODEIN PHOSPH. 20 MGS.	NARCOTIN HCl 20 MGS.	NARCOTIN HCl 8 MGS.	NARCEIN HCl 10 MGS.	THEBAIN HCl 10 MGS.	NARCOPHIN 20 MGS.	NARCOPHIN 10 MGS.	NARCOPHIN 5 MGS.	PANTOPON 10 MGS.
	C.G.S. units	C.G.S. units	G.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units	C.G.S. units
HAND												
Before injection....	2553	4554	2829	2208	2484	5520	6624	7038	5451	3174	3588	4899
After injection.....	4209	4554	6210	2967	3036	5106	6210	7038	17664	5106	4209	6624
TONGUE												
Before injection....	1588	1897	1173	2208	2277	3036	2760	2622	1622	1725	1656	2070
After injection.....	3174	1794	2277	3036	2553	2760	2622	2553	3626	2829	2208	3588
LIP												
Before injection....	1002	1312	639	966	1104	2070	1312	2139	829	932	897	1156
After injection.....	2208	1380	1656	1346	1588	1794	1312	2070	4345	1139	1151	1725
NOSE												
Before injection....	1156	1002	518	1588	1244	1244	1622	1794	1156	1156	1036	1622
After injection.....	2277	1070	2484	1897	1656	1104	1520	1794	3450	2070	1380	2622

tion of the existence of undoubted cases of persons, who are not relieved by morphin, but are rendered even more sensitive by it.

Action of Combination of Alkaloids.—After a study of the individual alkaloids, the action of combinations of morphin and narcotin meconates (narcophin) and other salts were studied. It was found that a given dose of narcophin has greater analgesic power than is represented by the arithmetical sum of the effect of its constituents, morphin and narcotin. Thus 5 mgs. of narcophin produces distinct analgesia, while 5 mgs. of morphin alone, or 10 mgs. of narcotin alone, produces no such effect. This observation is in complete agreement with Straub's[7] views on synergism of these two substances. A similar action was observed with a mixture of the total opium alkaloids (Pantopon).

The accompanying table illustrates the maximal effects of the various drugs used on one of us, as expressed by the quantity of stimulus in C. G. S. units required to produce the threshold sensation of pain.

Although by the above described method only cutaneous sensations could be studied, we think that our observations are of some value in the study of the very important subject of analgesia. This research has been endowed in part by a grant from the Council on Pharmacy and Chemistry of the American Medical Association; the complete data with many tables will appear in the *Journal of Pharmacology and Experimental Therapeutics*, Vol. 7, No. 5, January, 1916.

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THE SURFACE-TENSION AT THE INTERFACE BETWEEN TWO LIQUIDS

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While working with Haber upon a theory of muscular motion it was found by Harkins that the capillary-tube method for the determination of surface-tension is very inaccurate whenever a basic solution is used. This method is also extremely sensitive to the action of dust particles and to the presence of certain impurities, since the surface involved in the measurement is very small. Of the other available methods the two best seem to be the measurement of surface waves and the determination of the weight of a falling drop. Of these two the former requires a very elaborate and expensive apparatus if the determinations are to be made with considerable accuracy, while on the other hand the drop-weight method makes use of comparatively simple apparatus and gives results which are reproducible with considerable accuracy.

The most complete treatment of the mathematical theory of the relation between the forms of drops and surface-tension is given in a book published in 1883 by Bashforth and Adams.[1] Much later than this,

in 1906, Lohnstein[2] applied the general theory to the special case of the hanging drop just before its fall and the residue left after its fall, and from the difference he obtained the magnitude of the falling drop itself. The equation which he used for the relation between the weight (W) of the falling drop and the surface-tension α is

$$W = 2\pi r \alpha f\left(\frac{r}{a}\right), \qquad (1)$$

where r is the radius of the tip and a is the square root of the capillary-constant, and $f(r/a)$ is a function of r/a. Since $f(r/a)$ varies from 1.0 to 0.6, it is evident that neglect of this correction may cause errors as great as 40% in the surface-tension. It seems somewhat remarkable that in work with this method the correction has been neglected almost as often as it has been applied. Lohnstein determined the values of the function of r/a for different values of r/a, and his calculation of these corrections from a theoretical standpoint would seem to make it possible to use the drop-weight method as an independent method for the determination of surface-tension.

Unfortunately, however, when it becomes desirable to determine surface-tension, it is found that Lohnstein did not carry his calculations to a sufficient degree of accuracy to make this method available for determinations where an accuracy greater than 4% is desired. It is therefore important that this correction, which is a very fundamental one in work on capillarity, should be determined with a greater degree of exactness. Forms of apparatus devised by the writers seemed to make it possible to determine this correction experimentally under conditions which are more ideal than is possible than when usual methods are used. Thus the substitution of experiments on the liquid-liquid interface for the ordinary method in which a liquid-air interface is used makes it possible to compare the drop-weight results with those obtained in a capillary tube of large bore (1.5 mm.) when the capillary rise is great enough (150 mm.) to give accurate measurements. Not only can the diameter of the large capillary be determined more accurately than that of the smaller tube used for measurements on a single liquid, but in addition it is much easier in the large tube to keep the surface of the meniscus in a pure condition. Then in the drop-weight determination itself the drop falls very much more slowly when it falls into a liquid than when it falls into a gas, so that there is less disturbance in the drop at the time of fall than when it breaks away at a relatively high speed. The corrections determined in this way are given in Table 1, and are shown in the form of a curve in figure 1. In the figure the ordinates

represent the values of the function of r/a, and the abscissae give the values of r/a. The complete circles represent Lohnstein's theoretically determined values, while the circles given in outline represent the new values determined experimentally. The figure shows that the experimental values give with Lohnstein's first two points for small values of r/a a much smoother curve than is given by his own theoretical values. The experimental values were determined by the use of a number of different liquids, and measurements were made both upon liquid interfaces and at the surface of a single liquid.

Morgan, who has carried out an extensive series of investigations[3] on the drop-weight method, does not use the general Lohnstein rela-

TABLE I.

Experimental determination of the values of the function f (r/a)

Point No.	r/a	$f(r/a)$	*Interface*	*Temperature degrees*
1	0.281	0.709	Water: Ethyl Carbonate	25
2	0.366	0.685	Water: Benzene	25
3	0.441	0.672	Water: Dimethylaniline	25
4	0.484	0.654	Water: Ethyl Carbonate	25
5	0.592	0.639	Water: Benzene	10
6	0.621	0.636	Water: Benzene	20
7	0.633	0.632	Water: Xylene	25
8	0.636	0.634	Water: Benzene	25
9	0.648	0.634	Water: Benzene	30
10	0.649	0.632	Water: Toluene	25
11	0.709	0.620	Water: Air	25
12	0.837	0.615	Water: Hexane	25
13	0.845	0.616	Aqueous solution of Sodium Chloride: Benzene	25 25
14	1.071	0.612	Benzene: Air	25
15	1.387	0.620	Aqueous Solution of Strontium Bromide: Hexane	25

tion, but considers that the law of Tate holds for drops which have a 'normal' form. This law he expresses in the form: 'Surface-tension = Constant × Drop Weight.' The use of this equation is equivalent to the assumption that in figure 1 the curve at the bottom is coincident with its horizontal tangent. From the form of the curve it may be seen that this is not strictly true at any point, but that no serious error is involved in this rule if the determinations of surface-tension are always made with a tip which gives a value of r/a very nearly that at which the tangent touches the curve. Usually the tube used does not meet this very specific requirement, and the result is therefore different from the true result by the distance between this tangent and the curve.

The results of Morgan and McCann[4] upon 5 different liquids with 16 different tips have been used to calculate the values of the function

$f(r/a)$. Of these only the results obtained on benzene have been plotted in figure 1. This curve is higher than that obtained by us, which means that their values for the surface-tension are less than ours. This, however, does not mean that the drop-weight results given by Morgan and McCann are incorrect, but that the capillary-constant for benzene which they use, and which they do not themselves determine, is lower than that found by us. They use an average constant taken from the results of Ramsay and Shields and other workers. Since the reading of this paper the supposition that these workers have in general obtained too low a capillary-constant has been confirmed by Richards and Coombs,[5] who considers that the results of Ramsay and

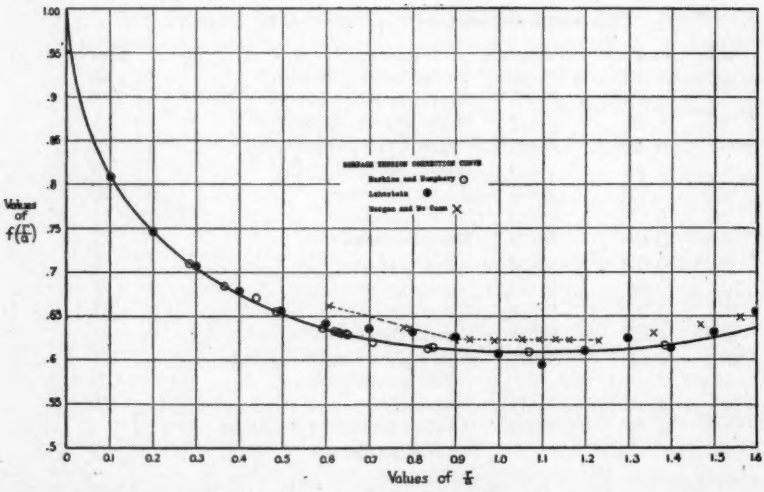

FIG. 1.—(NOTE) THE CURVE HAS BEEN DETERMINED ACCURATELY FOR VALUES OF $\frac{r}{a}$ UP TO 0.7. FOR VALUES GREATER THAN THIS ONLY THE GENERAL FORM OF THE CURVE HAS BEEN DETERMINED.

Shields, Renard and Guye, and Walden and Swinne, are too low, and that the discrepancy is of the order of 3%.

For values of r/a between 0.9 and 1.2 Morgan and McCann get a straight line for the values of the function, which would mean that Tate's law holds. Our determinations in this region have not yet been made with any great accuracy, since most of the work up to the present time has been on values of r/a between 0.3 and 0.7; but at least it can be easily seen that the general form of their curve is very different from ours.

This improved drop-weight method has been applied by us to the determination of the surface-tension at the interface between two liquids, and especially to the problem of the change of surface-tension at such an interface as that between benzene and water when the reaction of the aqueous phase is changed from acid to basic. This problem has been investigated to some extent by von Lerch,[6] who used the capillary-tube method and thus obtained very poor results.

The problem just referred to has an important bearing on the mechanism of muscular action. Two important facts have been established in regard to the motion of the muscles: first, that the active part of the muscle is always electrically negative to the part at rest, and second, that the active muscle shows an acid reaction. It has been shown by Hill that the amount of energy set free during contraction is directly proportional to the length of the fibrilles, and therefore to the area of their surfaces. According to Bernstein the force of contraction produced by a stimulus has a negative temperature-coefficient. All of these facts seem to suggest that the origin of muscular motion should be sought for in some form of surface energy. This was suggested as early as 1878 by Fitzgerald, who considered that changes of surface-tension are responsible for the phenomenon.

Haber and Klemensiewicz investigated the problem from a physico-chemical standpoint. It may be considered that the fibrilles of the muscles form one phase and the sarcoplasma another phase of a two-phase system. A similar two-phase system was constructed by the use of the interface water-benzene, and it was found that the change of reaction of the aqueous phase from basic to acidic caused a very rapid variation of electromotive force close to the neutral point. The change was found to be of the order of 0.5 of a volt. In similar systems von Lerch had found very rapid changes of surface-tension at the neutral point, so that if his results were reliable there would be a very good physico-chemical basis for the surface-tension theory of muscular action. Such a theory would be that a chemical reaction in the active muscle, such as the production of lactic acid or possibly of carbonic acid, causes the sarcoplasma to becomes acid, and thus changes the electromotive force, which in turn changes the surface-tension and causes muscular motion.

Unfortunately for this explanation our work indicates that the magnitude of the change of surface-tension at the neutral point is not so great as that found by von Lerch. The data determined by the writers will be presented in later papers to be published in the January (1916) number of the *Journal of the American Chemical Society*.

This research has been supported by a grant of three hundred dollars from the Warren Fund of the American Academy of Arts and Sciences, and for this aid we wish to express our indebtedness.

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OUTLINES OF A PROPOSED SYSTEM OF CLASSIFICATION OF THE NEBULAE BY MEANS OF THEIR SPECTRA*

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Presented to the Academy, October 22, 1915

This paper may properly be considered a continuation of one previously published in these PROCEEDINGS, 1, 266 (1915). In that article the opinion is expressed that the behavior of the line 4686A might serve as the basis of a classification of the nebulae. The notion will be discussed here in greater detail. About eleven of the planetary nebulae have been studied with a fair degree of completeness, and while this number is too low to afford the most secure basis for broad generalizations, the observations seem to point the way to a rational system of classification of these objects on the basis of their spectra.

In figure 1 are reproduced the spectra of nine planetary nebulae and the great nebula in Orion. As stated in the earlier paper the method adopted for observing the spectra of such nebulae consists in placing the slit of the spectrograph directly across the image. In this way the length of a spectral line is made to furnish a measure of the extent of the occurrence of the emitting material in the nebula.

The first spectrum shown is that of N. G. C. 7027. This nebula consists of two nuclei, of unequal brightness, surrounded by fainter nebulosity. In photographing the spectrum the slit was placed in the line of the two nuclei. The spectrum shows the nebula to be unusually homogeneous. Some of the fainter lines appear to be short but that is probably merely the result of their faintness, as they are no shorter than the

* This and the following paper contains, in abbreviated form, the substance of one read before the eighteenth (Pacific Coast) meeting of the American Astronomical Society, under the title: *The spectra of the gaseous nebulae and some points of correspondence between them and other celestial spectra.*

strong lines appear with greatly reduced exposure. None of the lines exhibit any tendency to undue localization in the nuclei. The fourth strong line from the right is 4686A, whose origin has so engaged the attention of physicists in recent years. It is quite as long as lines of equal strength in other parts of the spectrum. Flanking this line on either side are groups of fainter lines which should be noted, as they play important parts in the discussion which is to follow.

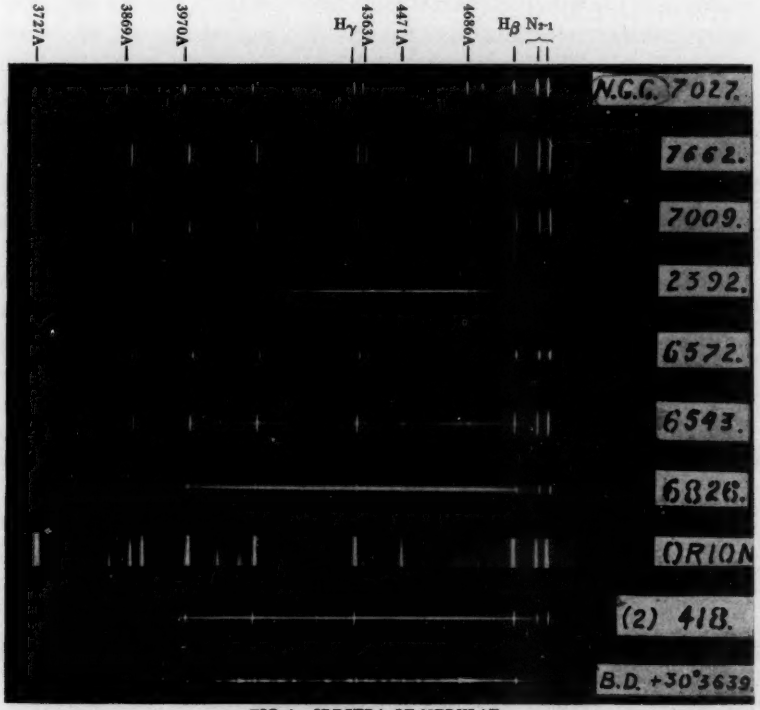

FIG. 1. SPECTRA OF NEBULAE.

The nebula N. G. C. 7662, whose spectrum follows, consists of a nucleus surrounded by an elliptical ring, the whole lying on a somewhat complicated background of fainter nebulosity. The increased brightness of most of the nebular lines where they cross the ring is plainly evident, particularly where the lines are not over-exposed. The line at wave-length 4686A and its neighbors are shorter than the others. They terminate sharply at the ring, and do not extend out into the faint outlying nebulosity and their brightness at contact with the ring itself is not greatly increased, which shows that they are relatively bright in

the comparatively dark area within. 4363A is longer than the members of this group but shorter than the hydrogen ($H\beta$, $H\gamma$, $H\delta$, $H\epsilon$) and nebulium (N_{2-1}) lines. Although the exposure extended over 17 hours the continuous spectrum of the nucleus is only faintly recorded, a fact which may be due, in part, to poor observing conditions.

Both in form and in spectrum the object N. G. C. 7009 bears a striking resemblance to 7662, and the purpose of exhibiting the spectrum is partly to call attention to this fact. Both of these spectra appear to bear a strong resemblance to that of the ring nebula in *Lyra* as described by Max Wolf, and it may be that in general objects of this form have a characteristic spectrum. The spectrum of the nucleus of N. G. C. 7009 is continuous, but the record is not strong enough to prove the presence or absence of dark lines.

N. G. C. 2392 is the nebula in *Gemini* which consists of two concentric rings of nebulosity surrounding a very bright nucleus. The usual nebular lines, including those due to hydrogen and helium, are present in the outlying nebulosity, but in the nucleus the hydrogen and helium lines are reversed, that is, they appear as dark lines. 4686A on the contrary is brightened and probably broadened on crossing the spectrum of the nucleus. One might expect, from this apparent tendency of the line to concentrate in the nucleus, to find it comparatively short in the nebula, but as a matter of fact it appears to be quite as long, in this object, as any of the other lines. In fixing the position of the spectrum in an order, the controlling factor of whose arrangement is the degree of concentration of the line 4686A, there is therefore some doubt as to which of these phenomena should control. As a nebular line the one in question is relatively longer in this nebula than in N. G. C. 7662, which should place it before that object, while its increased breadth in the nucleus would seem to indicate a position intermediate between N. G. C. 7009 and 6572. In the dilemma I have tentatively assigned it to the latter position. In addition to the dark lines referred to above, the ζ *Puppis* series is also dark in the spectrum of the nucleus.

The nebula N. G. C. 6572, or Struve 6, is a small object which brightens gradually to a hazy nucleus near the center. It will be noticed that the line 4686A is confined to the nucleus, and that it has completely lost its monochromatic character. It has the appearance of a hazy ball. The lines flanking it on either side are very short, and some of them are broadened. The lines of these groups become very active in the *novae* and some of them play important rôles in the Wolf-Rayet (Class O) spectra. The well-known lines 4634 and 4650A are among them. The spectrum of this nucleus has several of the characteristic

Wolf-Rayet bands, in addition to that at 4686A. One of these shows very faintly at 4057A. While the nucleus is undoubtedly to be considered a Class O star, I am in some doubt as to its exact position in the Harvard scale.

The spectrum of N. G. C. 6543 very much resembles the one immediately above it, except that the nebulous envelope is comparatively much larger. Five of the critical lines in the vicinity of 4686A are very short, while two of them: 4641 and 4713 are of medium length. The nucleus band, 4057A, visible only with difficulty in N. G. C. 6572, is strongly developed here. Unlike the one at 4686A, this band does not appear to have its counterpart in a true nebular spectrum, that is, it has not been observed except as a broad band confined to nebular nuclei, or other Class O stars. As in most of the other spectra the lines are of various lengths, helium being intermediate in extent between 4686A and hydrogen. The other lines fall in their usual order of length.

The following subject is the spectrum of the central region of N. G. C. 6826. The spectrum of the nucleus resembles that of the preceding object, but has the additional feature of interest that the nucleus band at 4686A, and probably the bright hydrogen lines, are accompanied on their more refrangible edges by dark lines in the spectrum of the nucleus. Dark ζ *Puppis* lines are also present, though faint, and H and K (Calcium) are also dark. In addition to 4686A, 4658 and perhaps 4650 occur as bright bands in the nucleus.

As has already been remarked, the nebula N. G. C. Index 418, situated in the southern part of the constellation of Orion, was found by Campbell to have a hydrogen atmosphere of greater extent than that of its 'nebulium atmosphere. The photograph confirms this result, and shows in addition the restricted occurrence of helium. The details of the nucleus spectrum are lost on this plate, through overexposure, so that it is necessary to state that 4650 and 4686A are present as broadened bands confined to the nucleus, the first named line being much the stronger. In the downward progression this line has developed from a comparatively insignificant nebular line into the strongest band in the spectrum of the nucleus.

The spectrum immediately below is that of the well-known Wolf-Rayet star B.D. $+30°3639$ which Campbell found to be surrounded by a hydrogen atmosphere. As announced a year ago it is in reality a planetary nebula with a highly complicated nucleus. The line 4686A and most of the neighboring lines which occur as nebular lines in N. G. C. 7027 are here represented by broad bands in the nucleus. Helium is in the nucleus, while hydrogen is still outstanding in the nebula.

The nebula N. G. C. 40, the spectrum of which is shown in figure 2, is a large dim ring with a comparatively bright nucleus. The spectrum was photographed with a very short camera, so the scale is small. The nebular spectrum is shown to consist principally of the hydrogen lines and the ultra-violet line at 3727A. As is usual in such cases the lines show knots at their extremities corresponding to the bright edges of the ring. N_1 and N_2 are present, though they are exceedingly faint, and appear to be short. The spectrum of the nucleus is over-exposed on this plate and is shown to better advantage, with reduced exposure, in figure 3. The original negatives of this spectrum are very small and the necessary enlargement has been so great that the definition is not of the best. It will be seen, however, that the spectrum of the nucleus is made up of numerous hazy bright bands, and measurement proves these to be well-known Wolf-Rayet radiations. There are seventeen of them in addition to the hydrogen bands. The brightest is 4650A, and the lesser one beside it is 4686A. If it is true that the nebulae

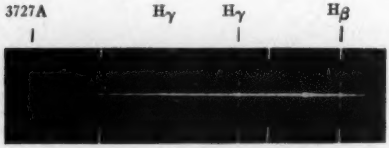

FIG. 2. SPECTRUM OF N.G.C. 40.

FIG. 3. SPECTRUM OF NUCLEUS N.G.C. 40.

are condensing into stars there are reasons for supposing that this one is further along in the process than any of the others we have studied.

In presenting these spectra an endeavor has been made to arrange them according to the degree of concentration of 4686A and some of the neighboring lines. It will be seen that the successive objects stand in very close relation to one another, yet at one end of the scale is a purely gaseous nebula, and at the other a banded star with only the shreds of nebulosity clinging to it.

Quite aside from any bearing which these observations may have on the subject of nebular evolution it appears to me that they offer the basis for a satisfactory classification of the nebulae. The observation of many of the nebulae for the purpose of investigating the degree of concentration of the lines which have been discussed would be impracticable but this, I believe, would hardly be necessary, for it will be observed that, having arranged the spectra in the order shown, there are certain progressions in the relative intensities of some of the comparatively bright lines which would serve to fix the positions of any of the spectra in the adopted scale. For instance, consider the doubtlet

at 3970A. The component to the right is H_ϵ, the one to the left is of unknown origin. These lines are so close that they are only incompletely resolved on some of the plates, a rough estimate of their relative intensities may, however, readily be made, and it will be seen that in the upper spectrum the left hand component is much the stronger. As we go down the line it loses steadily in strength until in N. G. C. 6543 and 6826 the two components are equal. In the Orion nebula, which has been inserted here, the left hand member is much the weaker of the two, while in the last two objects this component is practically missing. The same may be said of 3869A; it is strong in the upper pictures and finally fades out in the descent. In what appears to be a lesser degree there is a corresponding variation of the lines N_{1-2}; that is, the so-called chief nebular lines gradually weaken with reference to those of hydrogen. The line 3727A, on the other hand, undoubtedly strengthens in the downward progression, though this does not show very clearly in these plates by reason of the fact that the 36-inch refractor is an unsuitable instrument for studying the ultra-violet, the region of the spectrum in which these lines lie. The spectrum of the Orion nebula, in which this line shows so prominently, was made with the 12-inch telescope; aside from the undue strength of 3727A, which is to be accounted for by this fact, this spectrum is found to correspond closely with those on either side of it, and appears to be intermediate between them. There is therefore no essential difference between it and the spectra of the planetary nebulae which may be said to belong to the same class.

In the course of this work the spectra of nine planetary nebular nuclei have been examined. In two cases the observations so far made are not competent to determine with certainty the nature of the spectra. The remaining seven are undoubtedly Class O stars. I think it a fair inference that the nebulae and the Class O stars in general stand next to each other in the course of stellar evolution. If, as some astronomers believe, certain of the red stars stand next to the nebulae, it is remarkable that we do not find them associated with these objects. All of the nebular nuclei stand high on the temperature scale, if we may judge from their great photographic, as compared with their visual, brightness. So far as the development of stars from gaseous nebulae may be concerned the evidence seems to favor the usually accepted theory, rather than the red to red progression favored by Lockyer and some others.

SOME PROBABLE IDENTITIES IN WAVE-LENGTH IN NEBULAR AND STELLAR SPECTRA*

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Coincidence between the bright nebular lines and lines in the spectra of the stars and terrestial sources are usually regarded as limited to the radiations of hydrogen, helium, the line at wave-length 4686A, and the related series of ζ *Puppis* lines. There are other correspondences to which attention should be drawn. There is in the nebulae a very faint line close to Hδ which became pronounced enough in the spectrum of *Nova Geminorum* no. 2 to modify very perceptibly the form of the Hδ band. In fact it was first suspected in the *nova* from this circumstance, and afterward found, with prolonged exposure, in a number of nebulae. Its wave-length is uncertain on account of its faintness and the overpowering strength of the Hδ line which, with the low dispersion used, is very close. The mean of five measures gives 4097.6A. The wave-length of the dark line in ϵ *Orionis* is, according to R. H. Curtiss,[1] 4097.5A. This stellar line has been the subject of much discussion in studies of spectral classification. It has its maximum, according to Miss Cannon, in Class Oe stars. It has been assigned by various observers to as many elements, but the best case seems to have been made out by Lockyer, Baxendall and Butler who identify it with a line of abnormal behavior in the spectrum of nitrogen.[2] Whatever its chemical source its strength in Class O stars argues for its common origin in these and in the nebulae.

A line which plays an equally, or perhaps more, important part in stars a little further along in their development is 4267.30A. This is universally ascribed to carbon, and the wave-length quoted is Hartmann's laboratory determination made with a highly rarefied tube containing a gaseous hydrocarbon compound. A faint nebular line has been photographed in this vicinity by several observers, but only rough determinations of its position have been published. The mean of five of my measures places it at 4267.28A. The line is very faint and this value may be in error, say 0.2 of an angstrom. The South Kensington observers have found[3] the carbon line 4267.30 to be accompanied by a doublet in position 4647.53, 4650.92. If this doublet were in the nebulae it would not be resolved on these plates but would be represented by an apparently single line of slightly greater wave-length than

* See note on previous title, p. 590.

the mean, as the red component of the double is the stronger of the two. In other words a nebular line of slightly greater wave-length than 4649.2A would meet the requirements. There is actually such a line at 4649.5A. Both this and the one at 4267A probably exist as bright bands in the spectra of the Wolf-Rayet stars.[4]

The following is a summary of the wave-length comparisons which have been made:

	Star.	
Nebula	*Nitrogen*	*Carbon*
4097.6	4097.5	
4267.28 ±		4267.30
4649.5		4649.2 +

In considering the significance of these comparisons it should be borne in mind that the observations of the nebulae were made with a single prism spectrograph and that the lines are faint and difficult to measure. I think, however, that the evidence renders probable the presence in the nebulae of carbon and nitrogen, and fortifies the assumption of a close relationship between the nebulae and the early type stars.

There are other possible points of correspondence between nebular and stellar spectra which will be referred to in a more complete presentation of the subject.

[1] *Pub. Ast. Obs. Univ. Mich.*, 1, 120 (1915).

[2] *London, Proc. R. Soc.*, A, 82, 534 (1909).

[3] *Ibid.*

[4] A paper is just at hand by Mr. T. R. Merton, entitled: On a spectrum associated with carbon, in relation to the Wolf-Rayet stars, *Lond., Proc. R. Soc.*, A, 91, 498 (1915). In this the author makes the suggestion that the two Wolf-Rayet bands in question, with others, are due to carbon.

ENERGY TRANSFORMATIONS DURING HORIZONTAL WALKING

By Francis G. Benedict and Hans Murschhauser

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No form of muscular exercise enters so universally into the lives of all individuals as does horizontal walking, but most of the earlier researches on the energy transformations consider walking on a horizontal plane as incidental to or as a base-line for the work of ascent, particularly in connection with mountain climbing. From the fundamental contributions of Zuntz and Durig and their associates, it has been concluded that for an individual walking on a horizontal plane the energy required to move one kilogram, either of body-weight or of

superimposed load, one meter in a horizontal direction is equivalent to 0.55 gram-calorie. These workers likewise noted the distinct influence of increased velocity upon the energy requirement for the same amount of work.

Prior to a direct calorimetric study of the influence of walking in a horizontal direction, as well as the work of ascent and descent, the present study was made to elaborate the earlier researches on horizontal walking. A modified form of the universal respiration apparatus was employed and a specially designed treadmill. The factors measured were the oxygen consumption, the carbon-dioxide production, the respiration-rate, the distance walked, the number of steps taken by the subject, and the height to which the body of the subject was raised in the up and down motion of walking. The values for the resting metabo-

TABLE I

SUBJECT NO.	AGE	NUDE WEIGHT	HEIGHT	CARBON DIOXIDE PER MINUTE	OXYGEN PER MINUTE	HEAT-PRODUCTION PER MINUTE
	yrs.	*kilos.*	*cm.*	*cc.*	*cc.*	*cals.*
I.....	29	69.7	180	223	280	1.34
II.....	31	68.3	177	214	258	1.25

TABLE II

SUBJECT NO.	NO. OF PERIODS	WEIGHT WITH CLOTHING	AVERAGE RATE OF WALKING PER MINUTE	INCREASE IN HEAT OUTPUT OVER STANDING	HEAT OUTPUT PER HORIZONTAL KILOGRAMMETER
		kilos.	*meters*	*cals.*	*gm.-cals.*
I.....	53	73.10	75.9	2.81	0.507
II.....	57	71.45	71.5	2.52	0.493

lism as determined for both the lying and the standing relaxed positions were taken as base lines for comparison with the values obtained with the subject while walking. A few experiments were made when the subject was walking at a high rate of speed and likewise when running.

The preliminary observations were made on one subject by Dr. Carl Tigerstedt of Helsingfors during his short sojourn at the Nutrition Laboratory. A more extended investigation was carried out on a second subject with special emphasis upon change in velocity and the influence of food, including experiments with uncontrolled diet and diets containing a preponderance of protein, fat, or carbohydrate. A few experiments were prolonged for the purpose of studying the possible influence of fatigue.

The metabolism found for the standing relaxed position, with the subject in the post-absorptive condition, is given in table I, and for

walking at moderate speed, without food, in table II. It will be seen from the latter table that the average value found in 110 periods with these two subjects was, in round numbers, 0.5 gram-calorie.

The results obtained in experiments after a meal showed that the ingestion of food raised somewhat the resting metabolism but was without material effect upon the forward progression constant of 0.5 gram-calorie per horizontal kilogrammeter.

In the prolonged experiments without food, in one of which the subject walked 22 kilometers, successive periods showed very little, if any, change in the constant, thus suggesting the absence of a fatigue effect. Singularly enough the 22-kilometer experiment with food showed a distinctly lower constant than the comparable experiment without food on the preceding day.

TABLE III

METHOD OF PROGRESSION	NO. OF PERIODS	(a) AVERAGE DISTANCE PER MINUTE	(b) AVERAGE RAISING OF BODY PER MINUTE	(c) AVERAGE NUMBER OF STEPS PER MINUTE	(d) LENGTH OF STEP $\frac{100\ a}{c}$	(e) HEAT (COMPUTED) PER HORIZONTAL KILOGRAM-METER
Without food		*meters*	*meters*		*cms.*	*gm.-cals.*
Walking:						
Slow.....	57	71.5	2.94	111	64.4	0.493
Medium.....	6	106.3	5.87	131	81.1	0.585
Fast.....	7	144.1	7.75	152	94.8	0.932
Running.....	15	147.5	13.75	182	81.0	0.806

The influence on the constant of an increase in the rapidity of walking and particularly of the change in type of locomotion from walking to running is shown in table III. In calculating these values, the metabolism in the standing relaxed position was used for the basal metabolism. It will be seen that with increased velocity the height to which the body was raised, the number of steps, and the length of each step were all increased. The constant for the motion of forward progression was also increased in value, especially at the highest speed.

A more profound effect on all the factors of locomotion is noted when the change was made from walking to running. With essentially the same speed for each method of progression, the height to which the body was raised in running was nearly double that in walking; the number of steps was increased 20% but the length was correspondingly decreased. Of most significance is the decided fall of 15% in the value of the constant, i.e., from 0.932 gram-calorie for rapid walking to 0.806 gram-calorie for running. Since in running the body is lifted

much higher than in walking, this is surprising. On the other hand, in the walking experiments there was, as is customary with trained walkers, considerable arm motion which was absent in the running experiments. Basal experiments made while the subject was standing still but swinging the arms in essentially the same amplitude and rhythm as when walking showed a great increase in the resting metabolism. The use of this base line reduces the progression constant for walking to 0.780 gram-calorie with an average speed of 144 meters per minute. This debatable procedure seems to emphasize the fact that for the most economical transport of the body, with or without superimposed load, some type of gait which reduces to a minimum the elevation of the body and the extraneous arm motion is most desirable.

The details of this research are reported in Publication No. 231 of the Carnegie Institution of Washington.

THE PHYSIOLOGY OF THE NEW-BORN INFANT

By Francis G. Benedict and Fritz B. Talbot

NUTRITION LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, November 2, 1915

The physiology of the first week of life is of especial importance owing to the unique position of civilized woman who, alone of all mammals, is usually for several days so completely exhausted by labor as to make her existence and even that of her child absolutely dependent upon the ministrations of others. The insufficiency of the breast secretion, the loss in weight of the infant, which almost invariably occurs, and the too frequent but rarely needed complete substitution of bottle feeding for breast feeding make a study of the actual needs of the new-born infant of unusual interest.

In a research carried out by the Nutrition Laboratory, in which a respiration chamber was used, measurements were made of the carbon-dioxide output and the oxygen intake, and records were secured of the pulse-rate, respiration-rate, and body-temperature of 105 new-born infants. The observations often began before the infant was 1 hour old, the child being placed in the respiration chamber as soon as it had been bathed and dressed. The metabolism was then studied, frequently in 1-hour periods, for several consecutive hours. The primary object of the research was to determine the basal minimum metabolism of the infant with special reference to sex, weight, age, and length.

From the data obtained in these observations, the respiratory quotients were computed which indicated the nature of the principal sub-

stance burned in the body, i.e., fat or carbohydrate. A critical study of these respiratory quotients shows that the new-born infant was not supplied with a superfluous amount of body carbohydrate to burn during the period of insufficient breast secretion. Indeed, the predominance of fat combustion suggests the possible necessity of supplemental feeding of carbohydrates.

Select periods of muscular repose were obtained in the majority of cases which supplied data for computing the basal metabolism or minimum heat-production. This minimum heat-production, computed on the 24-hour basis, ranged from 95 to 193 calories, but as the body-weight likewise varied, the values per kilogram of body-weight were also computed. These ranged from 32 to 52 calories per kilogram of body-weight per 24 hours. Even on the basis of per square meter of body-surface, supposedly the best basis for comparison, the values ranged from 459 to 732 calories per square meter of body surface per 24 hours.

Of the numerous comparisons made none showed sufficient regularity even to suggest a physiological law, until the factors of age and length were considered. On the basis of age the figures showed that frequently the lowest as well as the highest values occurred during the first 24 hours of life and it was clear that there were profound disturbances of the heat-regulating mechanism during this period. After the first day considerable regularity appeared in the results. When length was also considered as a variant, all infants between 1½ and 6 days of age, 48 in number, showed that the heat-production per square meter of body-surface per 24 hours per centimeter of length was remarkably constant. The values computed on this basis rarely varied more than ± 6% from the average value of 12.65 calories, thus establishing a physiological constant for new-born infants.

The increase in the metabolism due to severe muscular exercise, such as lusty crying, averaged 65% for all infants but actually exceeded 100% in 10 instances, with a maximum increase in one case of 211%. As an evidence of the possible increase above basal metabolism in a normal, healthy, new-born infant this is worthy of special note.

The pulse-rate showed average values as follows for the first eight days: 112, 114, 116, 116, 116, 122, 119, and 126 beats per minute. A profound disturbance of the rectal temperature accompanied the bath following birth and this probably explains the great fluctuations in the heat values for the first day. Indeed, it is suggested that the bath be deferred for a day to permit the deficient heat-regulating mechanism a longer period of adjustment.

The probable daily maintenance requirement of energy for a new-

born infant, with the usual periods of activity, asleep, and awake, is estimated to be approximately 62 calories per kilogram of body-weight per 24 hours. This takes no account of the requirement for growth, which may be neglected in considering the energy requirement for the first week of life. The results of the research give opportunity for suggestions as to supplemental feeding and methods of conserving energy.

A detailed report of the investigation, together with a complete translation of the interesting article on the respiratory exchange of infants, published by Hasselbalch in 1904, is given in Publication No. 233 of the Carnegie Institution of Washington.

A COMPARISON OF METHODS FOR DETERMINING THE RESPIRATORY EXCHANGE OF MAN

By Thorne M. Carpenter

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Presented to the Academy, November 2, 1915

The principal methods of determining the respiratory exchange of man in short periods require the use either of a chamber in which the subject is confined or breathing appliances which are attached to the nose or the mouth or, as in the case of a mask, to the face. Apparatus with breathing appliances are of two classes. In one the subject breathes into and out of a closed current of air driven by a positive blower, the products of respiration being absorbed and oxygen being admitted. In the other type of apparatus valves are used to separate the currents of inspired and expired air and the latter is either measured by a meter or collected in a bag or a spirometer, a sample being taken and analyzed by means of a suitable gas-analysis apparatus.

Thus far no adequate comparison of these different types of apparatus has been made, the only attempt at such comparison being the compilation of results obtained with various respiration apparatus to show that these apparatus measured accurately the respiratory exchange. It therefore seemed desirable to compare the several types of apparatus for determining the respiratory exchange, with men as subjects and with as nearly identical conditions as possible.

The apparatus employed in this investigation were the following: Bed respiration calorimeter (chamber type with closed circuit); two forms of the Benedict universal respiration apparatus, i.e., tension-equalizer unit and spirometer unit (apparatus with breathing appliance, closed circuit type); Zuntz-Geppert apparatus (valves with meter and gas-analysis apparatus); Tissot apparatus (valves with an automatically counterpoised spirometer); Douglas apparatus (valves with rubber-lined cloth

Results of comparisons of the respiratory exchange obtained with different apparatus.

APPARATUS COMPARED	NUMBER OF		ARITHMETICAL AVERAGE OF ALL EXPERIMENTS					
	Expts.	Subjects	Carbon-dioxide output per min.	Oxygen absorbed per min.	Respiratory quotient	Pulse-rate per min.	Respiration-rate per min.	Ventilation per min.
			cc.	*cc.*				*liters*
Bed respiration calorimeter	36	16	190	223	0.850	58.5	15.0	
Tension-equalizer unit			185	227	0.815	59.5	13.2	
Spirometer unit	9	6	198	233	0.850	59.5	14.1	
Tension-equalizer unit			197	231	0.855	58.5	12.8	
Zuntz-Geppert	11	6	186	227	0.82	64.5	17.0	5.96
Tension-equalizer unit			190	224	0.85	63.0	15.9	
Zuntz-Geppert	22	9	176	220	0.80	58.5	12.3	4.45
Spirometer unit			182	219	0.83	58.5	12.5	4.76
Tissot	10	2	167	194	0.86	48.0	10.2	4.26
Tension-equalizer unit			165	193	0.855	47.0	10.1	
Tissot	17	7	192	242	0.795	60.5	13.9	5.00
Spirometer unit			190	233	0.815	60.5	12.4	4.96
Douglas	16	8	178	224	0.795	62.0	15.3	5.15
Spirometer unit			189	231	0.820	61.5	14.3	5.04
Mouth-breathing, Tension-equalizer unit	9	5	187	220	0.85	55.0	13.6	
Nose-breathing, Tension-equalizer unit			183	219	0.835	54.5	13.2	
Mouth-breathing, Spirometer unit	5	4	190	225	0.845	64.5	14.5	4.96
Nose-breathing, Spirometer unit			185	220	0.84	63.0	14.7	4.88
Mouth-breathing, Tissot	5	3	197	247	0.80	58.0	14.2	4.68
Nose-breathing, Tissot			191	246	0.775	57.5	15.0	4.87
Mask, Spirometer unit	5	3	201	253	0.80	62.5	15.3	6.16
Nosepieces, Spirometer unit			206	252	0.815	63.5	14.7	5.36
Mueller valves, Tissot spirometer	5	2	194	233	0.83	58.0	17.6	6.48
Spirometer unit			187	225	0.83	57.5	20.3	5.88
Mueller valves, Tissot spirometer	7	3	192	233	0.825	59.0	15.5	5.73
Tissot valves, Tissot spirometer			188	231	0.815	57.5	20.9	5.64
As normally used, Spirometer unit	13	6	191	238	0.80	63.0	11.6	4.71
With added dead space, Spirometer unit			197	239	0.825	64.5	11 8	6 07
Automatically counterpoised, Tissot spirometer	7	4	187	231	0.81	59.5	14.0	4.83
Approximately counterpoised, Tissot spirometer			188	229	0.82	59.0	15.1	4.91

bag); masks, rubber mouthpiece, glass and pneumatic nosepieces, Mueller valves, and Haldane gas-analysis apparatus (laboratory form and portable form).

The subjects were healthy young men, mostly medical students and laboratory assistants. The comparison of any two apparatus was made by determining the respiratory exchange of a subject with both apparatus on the same day, preferably in alternate periods. The subject was, in both tests, in the post-absorptive condition with as complete muscular repose as possible.

Measurements were made of the elimination of carbon dioxide and the absorption of oxygen from which the respiratory quotients were calculated. Records were also obtained of the average pulse-rate, the average respiration-rate, the total respiratory ventilation, and the volume per respiration. A record of the degree of muscular repose was secured by means of a special device. The accompanying table gives a general summary of the results obtained in the principal comparisons of apparatus and their modifications.

From a study of the details of the experiments, it is considered that all of the apparatus employed are suitable for the determination of the total carbon-dioxide elimination and the oxygen consumption and that the simplest and quickest method is the Benedict universal respiration apparatus. Of the apparatus with breathing appliances, the type involving the analysis of the expired air is considered the best for the determination of the respiratory quotient. The use of a rubber-lined cloth bag for collecting the expired air affords the most favorable opportunity for thoroughly mixing the portion collected, but care must be taken to select a bag which is not appreciably permeable to carbon dioxide. A spirometer for collecting the expired air is preferable to a meter, as all of the air can be collected and sampled. The Tissot valves were found to be the most reliable and efficient of the various kinds tested. In the majority of the experiments the respiratory exchange was the same irrespective of the kind of breathing appliances used. The Haldane gas-analysis apparatus is considered to be the best for the analysis of expired air but the analyses should be frequently controlled, using the constancy in composition of outdoor air as a basis.

Adequate control tests, in which such combustible materials as alcohol, ether, or other substances, whose composition is known, are advised as a means of checking the accuracy of respiration apparatus in general. A sufficient number of experiments with any given conditions and conservatism in the acceptance of results are also strongly recommended.

A detailed description of all of the apparatus used, the results of the individual experiments, and a critical discussion of the technique of determining the respiratory exchange of man in short periods are given in Publication No. 216 of the Carnegie Institution of Washington.

NEURO-MUSCULAR EFFECTS OF MODERATE DOSES OF ALCOHOL

By Raymond Dodge and Francis G. Benedict

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Presented to the Academy, November 2, 1915

In accordance with its widely distributed 'Tentative Plan'[1] the Nutrition Laboratory of the Carnegie Institution has organized and initiated an exhaustive experimental study of the physiological consequences of the ingestion of small doses of ethyl alcohol in man. The first year's work under the psychological part of that Plan was devoted to an investigation of the effects of alcohol on a selected group of interrelated processes covering the fundamental neural activities at various levels of the cerebro-spinal system, from the simplest reflexes of the lumbar cord to the most complex cortical arcs that we could accurately measure by available laboratory techniques.

The selection of the particular group of neuro-muscular processes for measurement was determined by the following experimental demands: (1) The systematic demand for coördinate data covering as many as possible of the fundamental psychophysiological operations. (2) The interpretative demand for the least possible inclusion of unknown and uncontrolled factors. (3) The practical demand for natural reaction forms which would be comparable in a large number of individuals without special practice, and would show relatively little practice effect as a result of the experimental repetition. (4) The technical demand for dependable quantitative methods of stimulation and registration.

Of the simple reflex arcs which are available for experimentation, the patellar reflex and the protective lid-reflex were chosen, chiefly because of their similar latency and the accuracy of their modern techniques. Our measurements of these reflexes include data concerning their latency, the extent of the muscle contraction, and the relative duration of the refractory phase. Of the more complex cortical arcs the following were selected: (1) eye-reactions to suddenly appearing peripheral stimuli, a thoroughly practiced phase of each individual's spatial adjustment; (2) speech reactions to visual word stimuli; and (3) free associations. The last two are characteristic phases of the individual's adjustment to his

social environment. The fundamental perseveration tendency was measured by the partial memorization of series of words. Sensory changes were indicated by Martin's Faradic-threshold measurements. Motor coördination was studied in the velocity and accuracy of eye-movements, and in the reciprocal innervation of the antagonistic muscles of the middle finger. Finally, pulse records (chiefly electrocardiograms from body leads) were taken either continuously or at homologous points in the various experimental processes.

The subjects consisted of two groups: a main group of college graduates who were very moderate users of alcohol, and a smaller group of out-patients of the Psychopathic Hospital who had been under treatment for delirium tremens.

Normal base lines in all cases included two normal experimental days for each subject and for each kind of experiment. One normal day came before and one after the experimental days on which alcohol was administered. In addition a 'normal of the day' was recorded for each experimental process on the days in which alcohol was given. Two alcohol doses were used, one containing approximately 30 cc. and the other 45 cc. of absolute alcohol.

All the measurements show more or less rhythmic and arrhythmic variations. In our statistical theory we assumed that, given a sufficiently large number of measurements, the normal rhythmic and arrhythmic variations will tend to compensate each other, leaving the average experimentally conditioned change relatively unaffected. In our data these average results show two particularly significant marks of reliability: (1) Similar processes are similarly affected and in similar degree. (2) In general the larger dose of alcohol shows the greater experimental effect.

The effect of alcohol was calculated in all cases by comparing the differences between the 'normals of the day' and subsequent periods on the normal and on the alcohol days. The greatest percentile effect was found in the reflexes. In the patellar reflex alcohol increased the latent time 10% while it decreased the amount of quadriceps thickening 46%. In the protective lid-reflex it increased the latent time 7% while it decreased the extent of lid movement 19%. It increased the latent time of the eye-reactions 5%; that of the speech reactions 3%. Memory and the free associations were only slightly affected. Sensitivity to Faradic stimulation decreased 14% after alcohol. The number of finger-movements decreased 9%; and the velocity of the eye-movements decreased 11% as a consequence of the ingestion of alcohol.

Quite in contrast to the general depression of the neuro-muscular

processes at all levels of the cerebro-spinal system was the effect of alcohol on the pulse-rate. Under all the experimental conditions alcohol produced a relative acceleration of the pulse. In only a few cases with the larger dose did this relative acceleration become a positive acceleration so that the pulse-rate in the periods subsequent to the ingestion of alcohol was faster than during the normal of the day; but in practically every instance alcohol prevented the regular pulse retardation that accompanied the successive periods of moderate mental and physical work on normal days. This relative acceleration averaged approximately 3%. While this effect is intrinsically small, its regularity, the large number of records, and its concomitant variation with the size of the dose give the results a high degree of probability.

The effort to determine which of the antagonistic heart-regulating mechanisms was responsible for the relative acceleration demanded an analysis of the pulse data. A comparison of the relative changes in the durations of systole and diastole (method of Hunt) led to a thoroughgoing consideration of the variations in the pulse cycles during the different experiments. The records show a consistent tendency of alcohol to decrease the mean variations amounting to an average of 19%. Since the rhythmic and arrhythmic pulse changes within the limits of our 12–second records could not have been conditioned by the relatively slow-acting accelerator, it seems necessary to regard the decrease of the mean variation after alcohol as caused by a decreased responsiveness of the inhibitor. There is some evidence in the records that this paralysis of the inhibitor is not an exclusive effect, and it is probable that various natural and experimental conditions might be found which would shift the preponderance of paralysis to the accelerating mechanism. The small amount of change, its purely relative character, and the probability that both regulating mechanisms are affected doubtless give the conditions for the confusion of the scientific traditions concerning the effect of alcohol on the pulse rate.

In conjunction with the pulse acceleration, the general neuro-muscular depression may be regarded as presumptive evidence of the effect of alcohol on organic efficiency. In none of our data is there any indication of a pure facilitation effect of alcohol. Contrary to the theory of Kraepelin, we not only found no facilitation of the motor processes, but the depression of their simplest forms in the finger and eye-movements seems to be one of the most characteristic effects of alcohol. Indeed it is exactly these effects that correlate most closely with the average of all the effects for the several subjects. Practically it seems to follow that these processes may serve as a readily accessible indicator

of individual susceptibility to alcohol. Theoretically it seems to follow that the effect on the motor coördinations indicates a central tendency of alcohol.

A full presentation and discussion of the various techniques and the resulting measurements are given in Publication No. 232 of the Carnegie Institution of Washington.

[1] Tentative plan for a proposed investigation into the physiological action of ethyl alcohol in man. Privately printed and distributed January 1, 1913.

VARIATION AND INHERITANCE IN ABNORMALITIES OCCURRING AFTER CONJUGATION IN PARAMECIUM CAUDATUM

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In view of the difficulty of interpreting from the standpoint of evolution the changes that occur in the complex phenomena of inheritance from two parents, the study of reproduction from a single parent becomes of great importance. One of the difficulties in such work is that the lower organisms, in which uniparental reproduction commonly occurs, present relatively few characteristics that are at the same time variable and hereditary, among closely related individuals; though this is a condition demanded for studies of heredity and evolution.

Certain abnormalities that appear after conjugation in the infusorian Paramecium appeared to offer a favorable opportunity in this respect. These abnormalities vary greatly in occurrence, character, and degree; at the same time they are partially heritable. This paper is a summary of an extended study of these abnormalities in relation to the problems of inheritance, variation, and racial change.

Among the progeny of a large proportion (from 36 to 81% in different experiments) of exconjugants of Paramecium caudatum, abnormalities appear frequently. These abnormalities consist of irregularities in body form and dimensions, of many diverse types. Among them are monsters due to partial or irregular fusion; single bodies some larger, some smaller than normal; and a great variety of abnormal shapes. These animals propagate by fission; some of the lines of individuals thus derived from the exconjugants are quite without abnormalities, in others under the same conditions the abnormalities reappear for generations. Thus the abnormalities are hereditary. But diversities appear also within the abnormal lines themselves. Some of the indi-

viduals of an abnormal line may be normal, others abnormal in various degrees. Different lines show diverse proportions of abnormal individuals. It thus becomes possible to test by selection the inheritance of degree and kind of abnormality within a single stock all derived by fission from a single parent.

In long continued uniparental reproduction of any sort, a remarkable constancy in hereditary characteristics has been generally reported. All the progeny thus coming from a single parent have seemed uniform in their hereditary characteristics, though they may differ in their bodily appearance. And this is quite in agreement with the known cytological processes accompanying the two types of reproduction. In biparental reproduction there is a reduction and recombination of the nuclear elements, of precisely the same sort as the variation and recombinations of characteristics in the progeny, in Mendelian inheritance. In uniparental reproduction, particularly of the vegetative kind, such nuclear reductions and recombinations are not known; and the uniformity of the progeny is in agreement with this. These relations, with others not necessary to recount here, have given origin to the conception of the *genotype*, as the hereditary constitution, in contradistinction to the bodily appearance. The genotype is commonly held not to change in vegetative reproduction, or but rarely, and then by marked sudden steps, or mutations. In biparental reproduction the genotype does indeed change, but seemingly by mere shiftings and recombinations, in numerically predictable ways; so that the relations here are quite in agreement with the condition sketched above for uniparental reproduction.

A somewhat rigid, stereotyped scheme of heredity naturally results from the view of the facts just set forth: in particular, evolution by gradual change, guided by natural selection, appears to be excluded. This becomes still more marked if we conclude with Bateson ('14) that all mutations consist in the dropping out of factors. On the other hand, certain investigators in genetics oppose strongly this rigid view, holding that, over and beyond Mendelian recombinations, hereditary variations of slight degree are frequently occurring, so that evolution may well be continuous and guided by selection. The recent papers of Castle give typical expression to this point of view.

If hereditary variations are frequently occurring, aside from Mendelian recombinations, it should be possible to find them in vegetative reproduction. Here we are freed from the mixing of types which makes these relations so difficult to interpret in biparental reproduction. The abnormalities in Paramecium were, therefore, studied mainly with rela-

tion to this question: Can we by continued selection of normal individuals on the one hand, of abnormal individuals on the other, break our single stock into two or more, differing in hereditary constitution?

The results of this study are as follows:

1. The diversities in abnormality occurring within a single line of descent (derived from a single exconjugant) are in some lines not hereditary, so far as can be determined by long continued selection. In a very large proportion of the races in which the abnormals were regularly discarded and only normals retained to carry on the race, the abnormal character persistently reappeared, the selected normals producing abnormal progeny. In all the abnormal races there is a wide variation in degree of abnormality of the individual, from those perfectly normal to the monsters so deformed that they would never be recognized as Paramecia if their history were not known. Yet, as stated above, in most cases the progeny of all these variations were alike, the daughter cells of normal individuals being often just as abnormal, or even more so, than the daughter cells of monsters. This of course agrees with the conditions found in most of the studies on inheritance in 'pure lines' or clones; the diversities within the lines are not inherited.

2. But in other lines, diversities within the line showed themselves to be heritable, so that selection gave very different results from those usually obtained in pure line work. By selection, single lines, derived by fission from a single parent, were divided into two or more races differing hereditarily. This was successfully accomplished in twenty-five races; from each of these were isolated two lines, one quite normal, the other continually producing abnormalities,—the two cultivated side by side.

Calkins and Gregory[1] have in some cases obtained four diverse races from the four primary daughter cells, or 'quadrants' of an exconjugant —these being the four individuals that receive the four macronuclei produced before fission occurs. It is to be noted that our selection resulting in the isolation of lines differing hereditarily in abnormality often has been brought about much later in the series of generations, so that the differentiation has often occurred within the compass of a single 'quadrant,' or indeed within a much narrower fraction of the descent. In several cases, differentiation through selection did not begin till after several weeks had passed, with production of a great number of generations. Thus the results of selection in the present case cannot be interpreted as due to a primary difference in the four original macronuclei produced during conjugation. Selection is effective when begun with progeny of a single individual that has appeared many generations after conjugation.

3. In a race of Paramecium which upon extended examination shows no hereditary abnormalities, conjugation results in the appearance of many lines which are hereditarily abnormal, others which are normal throughout.

4. In the diverse lines descended from the separate exconjugants of a conjugating culture, the two lines descended from the two individuals that have conjugated together tend to be alike in respect to normality or abnormality. That is, if the progeny of the exconjugant *a* are abnormal, the progeny of its mate *b* are more frequently abnormal than would be the case if the distribution of abnormal races were not affected by conjugation.

Our main result, therefore, is that in respect to these abnormalities, while some lines are constant in hereditary character, in others hereditable variations do occur within the line, so that by selection it is possible to break the single stock into a number of stocks differing hereditarily. The genotype in these cases therefore does not remain constant in uniparental reproduction. The condition on which evolution through selection depends is therefore realized in respect to these characters.

The complete paper of which this a summary appears in the *Journal of Experimental Zoölogy*.

[1] Calkins, G. N., and Gregory, L. H., Variations in the Progeny of a Single Exconjugant of Paramecium Caudatum, *J. Exp. Zoöl.*, **15**, 467–525 (1913).

THE INFLUENCE OF THE MARGINAL SENSE ORGANS ON FUNCTIONAL ACTIVITY IN *CASSIOPEA XAMACHANA*

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1. *The Influence of the Sense Organs on the Rate of Regeneration.* The conclusion of many of the earlier workers on the problems of regeneration that the nervous system had an important part to play in the processes of regeneration has been attacked by most of the more recent students of these problems. While Bardeen ('01), Herbst ('96), Goldstein ('04), Wolff ('95, '02) and others maintained that there was some direct influence of the nervous system, or some portion of it (sensory ganglia Herbst), and Child ('04) recognized the indirect influence of the nervous system, exhibited through muscular activity; Goldfarb ('09), Stockard ('09) *et al* have denied that there is even an indirect influence of the nervous system.

An especially favorable opportunity for studying this problem is

afforded by the Rhizostomous medusa *Cassiopea xamachana* the disks of which can be separated from the oral arms and kept in dishes of sea water for an indefinite period. Such disks retain their capacity for regeneration, while on account of their shape and the location of the marginal sense-organs they permit of many types of operation which are possible only on some simple organism.

In order to determine the influence of the nervous system on the rate of regeneration the following experiments were carried out. Pairs of entire disks of the same size were compared from one of which all of the rhopalia were removed while from the other an equal amount of tissue was removed from the bell margin between the rhopalia. In these, as in all of the following experiments, the amount of regeneration was measured inward from the edge of a hole in the center of the disk from which a circular piece of tissue had been removed.

The active member of any pair of such disks, being compared, regenerated faster in 75% of the experiments. In 10% of the experiments the two disks regenerated at an equal rate, while in the remaining 15% the inactive member of the pair showed the higher rate of regeneration, thus confirming the observations of Zeleny ('07) and Stockard ('09) that under such conditions sometimes the active and sometimes the inactive member of any pair of disks regenerates faster.

The marked divergency among the results of this series of experiments indicated that there must be individual physiological differences among the medusa disks of sufficient amount to invalidate the conclusions drawn from the comparison of entire disks. In all of the later experiments, therefore, the two halves of *one and the same disk* were compared with one another. This comparison is made possible through the fact that in *Cassiopea* the muscles and nerve fibres are confined to the subumbrella surface of the disk so that when two narrow strips of the subumbrella ectoderm, on opposite sides of the disk, are removed the two halves are insulated from one another until a new layer of tissue containing nerve fibres has been regenerated over one of the denuded areas. About thirty-six hours would elapse before functional nerve fibres would be regenerated, so by scraping over the strips each day the insulation of the halves of a disk was readily maintained.

In the second type of experiments the two halves of a disk were insulated in the manner just described; the sense-organs were removed from one half while an equal amount of tissue was removed from between the sense-organs of the other half. The disks were then allowed to regenerate in jars of normal sea water, the insulation between the halves being maintained throughout the experiment. The physiologi-

cal characteristics of the two halves of any disk would at the start of an experiment be identical, and the pulsations of the half on which the rhopalia remained were sufficiently strong to move the inactive half about in the water and thus reduce to a very small amount any difference in aëration.

In every instance where the rates of regeneration of the two halves of a medusa disk under these experimental conditions were compared it was found that the active half—*i.e.*, the half on which the rhopalia remained—regenerated at a more rapid rate than the inactive half. The results of two such series of experiments in which the insulated halves of 40 disks were compared are shown in curves *A* and *B*, figure 1.

The proportion between the amounts of tissue regenerated each day, when that from the inactive half is taken as the unit was respectively: first day, 1 : 1.53; second day, 1 : 1.44; third day, 1 : 1.41; fourth day, 1 : 1.38; and for the fifth day, 1 : 1.39.

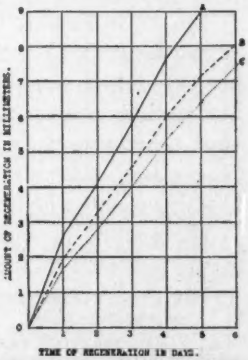

FIG. 1.—COMPARATIVE RATES OF REGENERATION OF SPECIMENS IN SEA WATER. *A*. HALF-DISKS WITH SENSE ORGANS ATTACHED. *B*. HALF-DISKS ACTIVATED WITH CIRCUIT WAVE OF CONTRACTION. *C*. HALF-DISKS WITHOUT SENSE ORGANS (INACTIVE).

Although the results from the experiments just described show conclusively that each half-disk on which the sense-organs were present regenerated faster than its companion half-disk without sense-organs, no evidence was afforded respecting the nature of the influencing factors. Since the muscular activity was the most striking difference between the two half-disks in this type of experiment other operations were undertaken to eliminate this factor. When disks with insulated active and inactive halves are put into a solution made up of 85 parts sea water and 15 parts 0.6 *m* $MgSO_4$ the activity first of the muscles, then of the rhopalia, and finally of the conducting portion of the nervous system is entirely suppressed. Under these experimental conditions the rate of regeneration of both half-disks becomes very much lowered. For the first few hours the half-disk on which the rhopalia remain regenerates faster, but after the full effect of the magnesium has become established the rate is the same for both halves.

As shown in the curves in figure 2 which represents the results of the comparison of the halves of 40 disks the difference in rate of regeneration is apparent only during the period before the rhopalia had come completely under the influence of the anesthetic.

These results, while confirming the previous observations that a half-disk under the control of the rhopalia regenerates faster than its companion half-disk from which these organs have been removed, were not sufficient, because of a lack of knowledge of the differential action of the magnesium ions on the several tissues involved, to determine whether the factor of muscular activity was or was not concerned in the initial difference in rate of regeneration. Fortunately another type of operation made known by Mayer ('06) makes it possible to determine the influence of muscular activity in a half-disk from which the rhopalia have been removed. When a series of cuts are made in the subumbrella tissues of a half-disk in such a manner that an endless labyrinth of tissue is formed, an entrapped wave of contraction can be initiated by an induction shock. This contraction wave will traverse the labyrinth of tissue for days before the muscles are overcome by fatigue so that the regeneration of new tissue to fill the open center of the disk will be completed before the contractions cease.

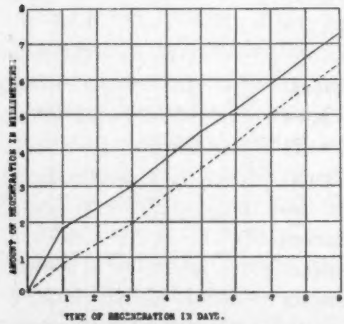

FIG. 2.—COMPARATIVE RATES OF REGENERATION OF SPECIMENS IN MAGNESIUM SOLUTION. SOLID LINE HALF-DISKS WITH SENSE ORGANS ATTACHED. BROKEN LINE HALF-DISKS WITHOUT SENSE ORGANS.

When the two insulated halves of a disk, from which all of the rhopalia have been removed while in the subumbrella tissues of one of them a circuit wave of contraction is maintained, are compared the actively contracting half-disk shows a slightly higher rate of regeneration than the inactive half. The difference in the rapidity of regeneration of the two halves in these experiments, as is shown by comparing curves *B* and *C*, figure 1, is not nearly as great as when a half-disk without rhopalia is compared with one on which the rhopalia remain (curves *A* and *C*, figure 1). When the fact that the rate of pulsation of a half-disk contracting under the influence of an entrapped wave is on the average more than three times the normal rate for a disk of the same size is taken into account it is at once apparent that some factor other than muscular activity must be the most important influence in determining the rate of regeneration. Added confirmation of this conclusion is obtained when the rates of regeneration of two insulated half-disks are compared, one of which is contracting normally under the influence of its rhopalia while an entrapped wave of contraction is maintained in the subumbrella tissues of the other (curves *A* and *B*,

figure 1). In experiments of this type the rate of regeneration is throughout its course about 1.3 times greater for the half-disk on which the rhopalia remain than for the half of which the muscles are contracting under the influence of a circuit wave of contraction. In terms of muscular activity the last mentioned half-disk is developing more than three times the energy of the former but its rate of regeneration still remains lower than that of the half-disk under the control of the rhopalia. It is quite apparent, therefore, that there is no direct relationship beween the extent of muscular activity and the rate of regeneration, but that some other factor directly related to and controlled by the marginal sense-organs plays the most important part in determining the rate of regeneration. In the absence of the influence of the sense-organs regeneration can take place in an apparently normal manner, but always at a decidedly lower rate than that shown by a specimen under the control of the sense-organs.

2. *Influence of the Sense-Organs of Cassiopea on the Rate of Metabolism as Measured by Production of Carbon Dioxide.* The striking evidence of some sort of influence exerted by the rhopalia on the rate of regeneration in *Cassiopea*, as described above, naturally raises the question as to whether or not a similar influence on other activities not included in the former experiments can be detected. Since a measure of general metal bolic activity may be obtained by determining the relative rates of CO_2 production of different animals under various sets of experimenta-conditions a series of determinations of total metabolic activity of half-disks of *Cassiopea* under conditions similar to those characterizing the regeneration experiments were carried out. To accomplish this end the disks were separated into halves, the appropriate operations performed, and each one put into a separate jar containing 1000 cc. of sea water. The jars were then tightly closed by a rubber gasket and a clamped top, and, after a given interval of time, the relative amount of CO_2 produced was measured by titrating against normal sea water taken as a control at the time the jars were filled.

When a half-disk with its sense-organs was compared with the other half of the same disk from which the sense-organs had been removed the amount of CO_2 produced was always found to be greater for the normal half-disk. This result is parallel to that obtained from the comparison of the rates of regeneration and may be, at least in part, explained by the activity of the muscular system in the half-disk with sense-organs. When a comparison of the rates of metabolism is made between the halves of a disk from which all its sense-organs have been removed while the muscular system of one-half is contracting under the influence

of an induced circuit wave of contraction the difference in the amounts of CO_2 produced is somewhat smaller than in the former experiment although the rate of pulsation in the activated half-disk is on the average 3.143 times as great as that of a half-disk pulsating under the control of its sense organs. It seems to be clearly established from this type of experiment that there is some other form of metabolic activity which is of greater importance as a source of CO_2 and which is more directly under the influence of the rhopalia than is the activity of the muscular system.

In the comparison of normally pulsating half-disks and those activated by a circuit wave of contraction a series of cuts, equal in extent to those used in forming the labyrinth in activated specimens, were made in the tissues of the half disks with sense organs in order to guard against any inaccuracy in the results due to inequality in the extent of the laceration of the tissues. The activity of the muscular tissue—rate of pulsation—was 3.143 times as great in the half-disk containing the circuit wave of contraction as in the normally contracting specimen, but after various periods of from 5 to 15 hours, when the titrations were made it was found that in every experiment except one the greater amount of CO_2 had been produced by the half disk on which the sense-organs remained. In the single experiment which proved an exception to the regular result the excess of CO_2 in favor of the activated specimen were very slight in comparison to the difference in rate of pulsation which for this specimen was 28 per minute for the half-disk with rhopalia and 118 per minute for the activated half-disk.

HERITABLE VARIATIONS AND THE RESULTS OF SELECTION IN THE FISSION RATE OF STYLONYCHIA PUSTULATA

By Austin Ralph Middleton

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In organisms in which an admixture of two parents occurs at reproduction, the problems of evolutionary change become most complex and difficult. Heritable changes appear abundantly, but most of these are shown to be the definite working out of the numerical rules of inheritance. Whether any of the heritable changes that occur are of a different character is in dispute. In reproduction from a single parent these difficulties disappear; if evolutionary changes occur independent of biparental admixture it should be possible to demonstrate this in uniparental reproduction. Yet most recent work agrees that in such uni-

parental reproduction inherited variations occur rarely or not at all, and that selection has no effect in altering racial characteristics.

Most investigators, following Johannsen ('03, '09, '11), have found that 'pure lines' or 'clones' are hereditarily constant under selection. Johannsen's results were obtained with self-fertilized lines of beans. Similar ineffectiveness of selection has been found by Hanel ('08) and Lashley ('15) as to the number of tentacles in Hydra multiplying by budding; by Jennings ('08, '09, '10) for size in infusoria; by Barber ('07) (in the main), and by Winslow and Walker ('09), in bacteria; by East ('10) in the vegetative reproduction of the potato; by Agar ('13 and '14) in Cladocera and aphids multiplying parthenogenetically; and by various other investigators on diverse organisms. Some discordant results have been recorded, but most of these are ill-defined or uncertain; it is mainly in bacteria, with their immense difficulties for precise technique in pedigree work, that heritable variations or modifications have been described. The preponderance of evidence has been that in uniparental reproduction heritable variations do not occur (save as rare mutations of marked character), and that selection of slight individual variations is without effect in altering the hereditary characteristics.

These results have given origin to the concept of the Genotype (Johannsen), as a designation for the permanent heritable constitution of the race. The present paper deals with the inheritance of variations and the effect of selection in the case of a most delicately poised and readily modifiable physiological character, the rate of fission of an Infusorian multiplying without conjugation.

The questions here raised are: Can we, with respect to the character examined, get from a single genotype by selection two genotypes that differ characteristically from each other under identical conditions; and that retain these differences from generation to generation? Is selection of small variations such as appear within the pure strain or clone an effective evolutionary procedure?

For one hundred and thirty days two halves of a single clone (or set of individuals derived from the fission of a single parent) of Stylonychia pustulata were subjected to selection in opposite directions; this produced a marked and steadily increasing difference in the average fission rate of the two halves. Expressing the excess of generations produced by the fast-selected lines as a percentage of the total number of generations produced by both sets, the difference was 6.9% for the first thirty days; 12.8% for the next twenty days; 19.3% for the next thirty days, and 21.2% for the last fifty days. The records also show that for the fast lines the number of generations produced per line during the one

hundred and thirty days ranges from 178 to 187, while for the slow lines the range is but from 116 to 128. The slowest fast-selected line produced 50 more generations than the fastest slow-selected line.

To determine whether the difference in fission rate thus produced is heritable, parts of the two sets were removed at intervals and subjected to culture without selection (by 'balanced selection,' in which unavoidable selections in one direction were always compensated by equal numbers of selections in the other direction). By long continued culture without selection, it was found that the difference was heritable. For example, after the two halves of the clone had been subjected to continuous opposite selection for 80 days it was found that the average difference per line per day had increased from 0.267 generation for the first thirty days to 0.415 generation for the last thirty days of that period. To test the permanence of this result these two sets of thirty lines were now subjected to ninety days of balanced selection, or ten days longer than the lines had been subjected to opposite selection. It was found that the average difference per line per day in favor of the progeny of the fast-selected set was for the three consecutive thirty-day periods of this experiment: 0.213, 0.256 and 0.284 generation.

Also, representatives of the two sets after 80 days of selection and 40 days of no selection ('balanced selection'), were subjected to mass culture for twelve days. Further balanced selection of these for fifty days showed that the inherited difference of fission rate still persisted. Thus the inherited difference produced by 80 days of selection had lasted for 102 days without selection.

Experiments with reversed selection showed that the inherited difference could be reversed in the same way that it is produced; the originally fast set was thus caused to become the slower one, and vice versa. Continuation of these two sets without selection showed again that the difference so produced was heritable.

Thus in this case the selection of small individual variations in fission rate has split the single clone (derived vegetatively from a single parent) into two heritably diverse divisions and the effect of selection was cumulative.

This experiment was now twice repeated. The first repetition was with the progeny of a single individual taken from one of the fast lines of the first set of experiments. Here again selection produced from the progeny of a single individual two sets differing hereditarily in average fission rate. Again, a new wild individual was obtained from a new mass culture. From this, two sets of thirty individuals each, all belong-

ing to the seventh filial generation, were obtained; and subjected, in the manner previously described, one to 'fast' the other to 'slow' selection for thirty days. The excess of generations produced by the fast-selected lines, expressed as a percentage of the total number of generations produced by both sets, was 1.99% for the first ten days; 4.36% for the second ten days, and 7.10% for the third ten days; thus showing a gradual increase and indicating a cumulative effect of selection.

To test the permanence of this result these two sets of lines were now subjected to balanced selection for 21 days. On every day but one the lines that had been subjected to fast-selection averaged higher than the others, and the percentage that the difference in favor of the fast set is of the total number of generations produced by both together was practically constant; it was 3.97% for the first ten days and 5.01% for the last eleven days. The results are thus the same as in our first set of experiments.

It was found that if conjugation occurred in the fast set, and likewise in the slow set, the difference produced by selection continued to exist after conjugation.

This third series of experiments has entirely corroborated the results of the first and second series. In a second clone, unrelated to that used for the first and second series of experiments, opposite selection for thirty days produced a heritable difference of average fission rate, a difference that gradually increased as selection progressed, indicating again that the effect of selection on this physiological character is cumulative. This average difference persisted through twenty-one days of balanced selection, twenty-nine days of mass culture followed by conjugation, and then fifteen days of further balanced selection.

All the experiments thus give concordant results; through selection of individual differences in fission rate it is possible to divide a clone into two divisions differing hereditarily in rate of multiplication. The effects of selection are cumulative; the hereditary differences between the two divisions become greater the longer selection continues. By reversing the direction of selection the hereditary differences between the sets are reversed. The hereditary differences between the sets persist through the ordeal of conjugation.

Hence, in reference to the physiological character studied, the selection of small individual variations such as appear within the pure strain or clone is an effective evolutionary procedure.

How are we to account for the difference between the results here set forth and those of older investigators? In Stylonychia we are dealing with an organism which is large enough to be easily handled and

followed individually, so that no question can arise as to the purity of the pedigrees (as sometimes occurs with reference to Bacteria). In this organism the facts as to the cumulative effects of selection are clear.

We are of course dealing with a delicate physiological characteristic, and this is perhaps more readily varied (even hereditarily) than the characters examined by most other investigators. Further, it is perhaps true that hereditary changes are more easily brought about in the Protozoa than in the more complex organisms, for in Protozoa the 'apparatus of heredity' is in close chemical contact with all the somatoplasm.

But a certain feature of the experimental procedure in the present case may have more importance than these conjectural considerations. It has been possible in my work to make a much greater number of actual selections (where plus and minus cases were both present to choose from), than in most of the work that has given negative results. And it has been found that a few selections give very slight results, and that a great number are required to give any marked differences between the sets. Thus, in my main experiment, on the average 39.86 plus selections were made in the fast-selected lines; 34.36 minus selections in the slow-selected lines. The difference between the two sets was thus the equivalent of some 74 selections extending through an average of 150 generations. This resulted in the production of a constant average difference per line of 0.42 of one fission per day.

Contrast with this great number of selections the *six* made by Johannsen in obtaining his negative results with beans, the three or four made by East with potatoes, the two made by Winslow and Walker with bacteria, and similar small numbers made by most other investigators along these lines; even indeed the selection through fifteen generations made by Agar, in Cladocera. It appears not at all inconceivable that in these organisms an equal number of selections, covering as great a number of generations, as were made in Stylonychia, would have given similar heritable effects. What all the work shows (and here my own is not in positive disagreement) is that heritable variations of considerable extent do not occur so frequently as was at one time supposed, so that a few selections are not sufficient for establishing a definite positive effect. But negative results from a few selections are not sufficient for disproving the occurrence of heritable small variations which may be gradually accumulated. This indeed has been admitted by many of those who have obtained negative results.

As a result of this work upon Stylonychia it is possible to give pre-

cise data as to the occurrence of heritable variations and their accumulation through selection, when sufficiently long continued. And this can hardly fail to have influence on the conception of the hereditary constitution or *genotype* as a fixed thing, changing only discontinuously by marked steps or mutations, that do not intergrade.

The full paper appears in the *Journal of Experimental Zoölogy*.

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HEREDITARY ANCHYLOSIS OF THE PROXIMAL PHALANGEAL JOINTS (SYMPHALANGISM)

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There are many recognized forms of congenital malformations of the hands and feet. Walker in 1901 first described the type of deformity which is made the subject of this study, and showed that the condition had been transmitted through five generations, though the number of his recorded cases was too small to justify a definite conclusion on a Mendelian basis. Farabee in 1905, and Drinkwater in 1908, showed that another type of deformity of the hands, known as brachydactylism, was a dominant unit-character, transmitted in accordance with the Mendelian law.

The lesion in the condition under discussion consists of a congenital anchylosis, due apparently to the failure of formation of the joint between

the proximal and middle row of phalanges. One or more, often all of the fingers are involved. The resulting deformity is known in the community as 'stiff fingers,' in contradistinction to the normal which are called 'crooked fingers.' This condition has been transmitted through seven generations, the progenitor of the family having migrated from Scotland to Virginia in 1700. There are connections of the family still in Scotland who carry the trait.

In the Virginia branch, which has been made the object of this statistical study, record has been secured of 312 descendants, among whom there were 84 affected persons, a few more than the 25% of the total number which would be expected. Excluding the incomplete families of the first three generations, in which were recorded few other than the affected persons carrying the trait, there are 72 completed families comprising 302 individuals. Of these 72 completed families, 44 of them were from the mating of unaffected parents with 152 unaffected children. Of the 28 families in which there was an affected parent, there were 150 children, 78 of them, or 52%, carrying the trait. It has been observed that the trait may be transmitted in outspoken form by a parent in whom it is inconspicuous, though never by an unaffected parent. The trait moreover is transmissible by either sex, and both hands and feet of the affected individuals may be involved.

The character, in short, behaves as a simple Mendelian dominant, with equal chance, among the offspring of affected individuals, that it will be or will not be inherited.

This paper will appear in full with photographs and charts in a forthcoming number of *Genetics*.

THE RELATIVE STIMULATING EFFICIENCY OF SPECTRAL COLORS FOR THE LOWER ORGANISMS

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The relation between color and reactions in organisms has for many years been a prominent problem in the study of behavior. The earlier investigators (Bert, Lubbock, Romanes, Graber and others) were interested in this problem largely from the point of view of comparative psychology. Their aim was to ascertain the relation between color-vision in man and in the various animals with the hope of thus elucidating the evolution of psychic phenomena.

Loeb studied the reaction to colors in plants and in animals for the

purpose of showing that they are the same in both, hoping thus to demonstrate that there is nothing in the nature of psychic phenomena in animals. Other investigators, e.g., Engelmann, Wiesner, Strasburger, Verworn, Parker and Blaanw, were interested in the problem more from the point of view of comparative physiology.

I have for some time held the opinion that, aside from the importance of this problem in comparative psychology and physiology, it ought to yield results which would throw light on the nature of the chemical changes in the organisms associated with the reactions to light, especially those associated with changes in the sense of the reactions.

The chief difficulty encountered in the work has been connected with obtaining monochromatic light and measuring it in terms suited for comparison. Thus while a considerable number of organisms, both plants and animals, have been investigated in regard to the relation between wave-length and stimulation, only in a few are the results of such a nature that they can be compared with sufficient accuracy to warrant more than very tentative conclusions. Moreover, in a number of cases, otherwise excellent, only the region in the spectrum of maximum stimulation has been ascertained. In the following observations, to be published in full elsewhere, these difficulties and defects have been to a large extent eliminated.

It is well known that many of the simplest organisms respond very definitely to light. Some orient and travel fairly directly towards the light, others away from the light, while still others go toward it under some conditions and away from it under others.

In a field of light consisting of two horizontal beams crossing at right angles these organisms proceed toward or from a point situated between the two beams. The location of this point depends upon the relative effective illumination received by the organisms from these beams. If it is the same in quality and quantity, so that the stimulation is the same, the point lies approximately half way between them. (Details as to the process of orientation in these organisms may be found in my book *Light and the Behavior of Organisms*, John Wiley & Sons, New York, 1911.) Consequently whenever the organisms proceed toward or from a point thus located, it may be concluded that the stimulating effect of the light in the beams is equal, no matter how the light may differ, either in quantity or in quality. (This is literally true for only a few organisms, but the principle as applied holds for all, as will be demonstrated in the extended paper to follow.) It is, therefore, obvious that if the light in one beam is kept constant in quality, white for example, while that in the other is changed in color the relative stimulating

efficiency of the different colors can be ascertained. To do this all that is necessary is to vary the luminous intensity of the white light for each change in the colored light until, in each case, the organisms proceed on the same path. The stimulating effect of the different colors will then be directly proportional to the various luminous intensities of the white light required to make the organisms under each of the different conditions, proceed in the same direction, e.g., if for green it required twice as much light from the white source to make the creatures take a given course as it does for yellow, then the stimulating effect of the green is twice as great as that of the yellow. To ascertain the relative efficiency in terms of wave-lengths and energy it is only necessary to use a spectrum having a known distribution of energy, and to make corrections in accord with this distribution.

In the experiments referred to below, two gas-filled street-series tungsten lamps with coiled filaments were used in series to produce two beams of light. One of these beams passed through a Hilger constant deviation spectrometer and the other through a Lummer-Brodhun rotating sector. The whole apparatus was so arranged that the two beams of light crossed at right angles in the field of observation. For every color tested the intensity of the illumination in the beam of white light was adjusted by varying the opening in the sector, until the course of the organisms bisected the angle between the two beams of light. In nearly all cases the successive regions selected in the spectrum differed by 10 $\mu\mu$. In this way the relative stimulating effect for the different regions of the spectrum was ascertained in fifteen different species as follows: Chlamydomonas, Trachelomonas and Phacus, each one species; Euglena, five species; Panderina, Eudorina, Gonium and Spondylomorum, each one species; earthworms, Arenicola (larvae) and blowfly (larvae) each one species. All but the last three are green, microscopic organisms, relatively very simple in structure.

The results obtained will be stated in terms of relative stimulative effects of the different regions of the spectrum tested without corrections for the difference in the energy of these regions. They are, however, of such a nature that the corrections mentioned will not result in marked alterations. These corrections will appear in the final paper.

For all but one of the microscopic organisms the results fall into two groups. In the one group the region of stimulation begins in the blue near the violet, between 430 and 440 $\mu\mu$. From here toward the red end of the spectrum the stimulating efficiency rises, at first slowly and then rapidly, to a maximum in the green near the yellow, between 530 and 540 $\mu\mu$; then it falls, at first rapidly and later more and more slowly,

ending in the red at about 640 $\mu\mu$. In the other group the region of stimulation begins in the violet between 420 and 430 $\mu\mu$, only a short distance from the place where it begins in the first group. From here the efficiency rises very rapidly reaching a maximum in the blue between 480 and 490 $\mu\mu$. It then falls rapidly and ends in the green in the neighborhood of 520 $\mu\mu$. Three of the microscopic forms, Pandorina, Eudorina and Spondylomorum, belong to the first group, the rest to the second. To this group belong also Arenicola larvae and the earthworms. For the remaining microscopic form (Chlamydomonas) the maximum is in the green very near 510 $\mu\mu$; and for the blowfly larvae it is approximately at 520 $\mu\mu$. The distribution in the spectrum, of stimulating efficiency is, for this creature, essentially the same as the distribution of brightness for totally color-blind persons. No difference in the relative effect of the different wave-lengths was discovered in any given species under different conditions. It was the same for organisms collected in different regions at different periods of the day and tested under various conditions of illumination and temperature, and it was the same for negative and positive individuals.

These results show that stimulation in all of the organisms studied depends upon the wave-length of the light; that the stimulating efficiency is very much higher in certain regions of the spectrum than in others; but that the distribution of this in the spectrum differs greatly in certain organisms that are closely related in structure, e.g., Pandorina and Gonium, while it is essentially the same in others that are very different in structure, e.g., Euglena and earthworms. They show, moreover, that if the absorption throughout the spectrum is the same in the different organisms the chemical or physical changes associated with the reactions differ in some species which are closely related. And that the changes in the organism, whatever they may be, which cause changes in the sense of the reactions are not reversible; for if they were reversible one would expect the distribution of stimulating efficiency in the spectrum to differ in positive and negative specimens of the same species.

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THE MISSION RANGE, MONTANA

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The Mission Range, one of the smaller members of the Rocky Mountains in western Montana, believed to be composed of deformed rocks chiefly quartzites, has the appearance of a gently tilted and moderately dissected fault block, trending north and south and about 70 miles in length. The steeper face, probably representing the battered fault scarp, looks to the west. At the low northern end of the range, the gently undulating crest emerges from the surrounding plains which are about 3000 feet in altitude, and rises slowly southward, reaching a height of 9500 feet near its abrupt southern end, thus gaining a local relief of 6500 feet. The eastern side of the range is said to slope more gently than the western face. The nearly even crest and long eastern

slope suggest that the mountain mass is an upraised fragment of a former worn-down mountain region.

The western face of the range, which I saw during a Shaler Memorial study of another Montana problem in 1913, may be divided as in the above figure into three oblique belts by two nearly parallel, south-dipping planes, about 1000 feet apart. The middle belt includes smoothly rounded summits and large-textured, full-bodied, waste-covered spurs, between wide-spaced, steep-pitching, apparently consequent ravines of normal erosion. All the high-reaching valleys of the southern belt expand in cirques of local glaciation at their heads, and continue downwards in narrowing troughs with oversteepened walls. These features are best developed at the high southern end of the range; there the cirques are huge cliff-rimmed cavities, strongly expanded southward; the mountain crest is sharpened to Alpine arêtes between opposing cirque walls; the long troughs, encroaching broadly on the normal forms of the rounded spurs, reach the foot of the mountains; and terminal moraines advance a short distance upon the piedmont plain, sometimes enclosing a lake in their loop. More than a score of glacial cirques and troughs may be counted, but their strength diminishes as the range

crest lowers northward, and the northernmost cirques are of small dimensions, faintly developed, and reach only a few hundred feet down from the valley heads. The limiting plane which touches the lower end of the troughs rises slowly northward from the base of the range at the southern end and reaches the crest of the range north of mid-length.

The lower northern belt of the range shows innumerable bare crags, knobs, cliffs and ledges of small texture, due to scouring and plucking by the terminal portion of a broad and overwhelming glacier of Canadian orig'n. The limiting plane, marking the height of the invading glacier, touches the range crest about a quarter length from its low northern end and descends to the range base south of mid-length. The northern quarter of the range, lying entirely beneath this limiting plane, exhibits glaciated forms in a minutely irregular crest and a disorderly slope of scoured and plucked hills and hollows, all of less and less height northward, until the last visible knobs, deeply scored and channeled and more or less detached from one another, rise only 100 feet or so above the surrounding intermont plain of glacial gravels and silts, diversified by low morainic hills, which presumably conceals a farther northward extension of the range crest. The second quarter of the range shows similar small-textured forms up to the limiting plane, but above it the mountain crest and slope retain the simpler, large-textured forms of normal erosion, until, near mid-length, the valley heads begin to show the cirques of the high southern belt. Through this second quarter each normal valley is barred by a morainic embankment on the line of the limiting plane, and below the plane each spur is imperfectly truncated in a bold and rugged slope which presents a tumult of rocky cliffs and ledges, descending abruptly and without well-defined valley reentrants to the waters of Flathead Lake. The apex height of the rugged spur facets and the altitude of the somewhat lower morainic embankments decrease slowly and systematically southward; the facets become smaller and less continuous with one another; the embankments become longer, larger, and more nearly continuous, until they curve away from the range base and form a noble terminal moraine, 400 or 500 feet in height and a mile or more wide, which swings westward across the intermont depression and separates Flathead Lake on its northern concave side from a broad till plain of earlier glaciation on its southern convex side.

It is to the long and gradual southward rise of the mountain mass that the Mission Range owes the clear separation of features due to local glacial sculpture in the high southern belt from those due to general glacial sculpture in the low northern belt, by the oblique middle belt

of normal sculpture. As far as I have seen and read, the range is unique in this systematic tripartite arrangement of normally and glacially sculptured forms. A fuller account of the range will be prepared for the *Bulletin of the American Geographical Society.*

DEFINITION OF LIMIT IN GENERAL INTEGRAL ANALYSIS

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1. *General Analysis.* The problem of Science is the organization of and the study of the interrelations amongst the objects and phenomena of Nature. Analogous objects or phenomena are grouped into classes. In the progress of Science, with the discovery of new objects or phenomena or interrelations, the bases of classification initially of necessity superficial become more fundamental; thus in Physics, Electricity and Magnetism and later Light merge in Electromagnetism.

Mathematics with its source in Nature progresses in similar fashion. Hence, remembering that the objects or phenomena of Mathematics may be theories (doctrines), we may enunciate the following heuristic principle:

The existence of analogies between central features of various theories implies the existence of a more fundamental general theory embracing the special theories as particular instances and unifying them as to those central features.

After the development of such a general theory, the fact that the various theories are instances of the general theory implies as an obvious consequence (and accordingly eclipses in importance) the analogies between the central features of the various theories. In illustration of the heuristic principle may be adduced the theories of General Analysis mentioned below.

Analysis is the branch of Mathematics devoted to the classification of and the study of the interrelations amongst numerically valued functions. A (single-valued) function τ or $\tau(p)$ is a table (or rule or process) assigning to every element or member p of a certain class or range $\mathfrak{P}$ a definite element q of a certain class $\mathfrak{Q}$. It is numerically valued in case the functional values $q, = \tau(p)$, are numbers real or complex. Although not always numerical, the independent variable p of a function τ considered in a theory of Classical Analysis is always of specified nature; e.g., the variable p may be a curve or a numerically

valued function, $\tau(p)$ being the length of the curve p or the average value over its range of definition of the function p. However, the nature of the element p is not fully specified in Fréchet's theory[1] (1906) of sets of elements p of a class $\mathfrak{P}$ and of continuous functions on such sets. This general theory has as instances the theories for the linear continuum $\mathfrak{P}$ and for the n-dimensional space $\mathfrak{P}$ initiated by G. Cantor and for the class $\mathfrak{P}$ of continuous curves due especially to Arzelà. Fréchet conditions his class $\mathfrak{P}$ by a definite but undefined relation L (the concept of convergence of a sequence $\{p_n\} : p_1, p_2, \ldots, p_n, \ldots$, of elements p to an element p_0 of $\mathfrak{P}$ as a limit) possessing certain properties; these properties of the relation L are in the special theories immediate consequences of the current definitions of the relation L in those theories.

Those theories of Analysis in which at least certain of the functions involved are on a range $\mathfrak{P}$ of elements p of a nature not specified, or at least not fully specified, we may designate as theories of General Analysis. Thus, Fréchet's theory with basis $(\mathfrak{P}; L)$ is a theory of General Analysis, as is likewise my theory[2] (1910) of classes of functions on a general range $\mathfrak{P}$.

A general range $\mathfrak{P}$ is an arbitrary particular range $\mathfrak{P}$ with abstraction of its particular features, e.g., Fréchet's range $\mathfrak{P}$ with abstraction of the feature L. Properties of functions, classes of functions, etc., on or connected with a general range $\mathfrak{P}$ (whose definitions accordingly involve no particular features of the range $\mathfrak{P}$) are of 'general reference,' while others are of 'special reference.' Thus, relative to a linear interval $\mathfrak{P}$, the continuity of a function is of special reference, while the property of uniformity of convergence of a sequence of functions and the property of the class of all continuous functions that the limit of a uniformly convergent sequence of the class belongs to the class are properties of general reference.

2. *General Integral Analysis.* A theory of General Analysis involving a numerically valued single-valued functional operation J (of the type of definite integration) on a class $\mathfrak{K}$ of functions κ whose range of definition involves a general range $\mathfrak{P}$ we may designate as a theory of General Integral Analysis. Thus, my theory[3] of linear integral equations is a theory of General Integral Analysis. This general theory has as instances the classical theory due to Fredholm and Hilbert-Schmidt for the case of continuous functions, and other classical theories.

3. *Definition of Limit in General Integral Analysis.* In a subsequent note I shall indicate a general theory of linear integral equations having as instance Hilbert's theory of functions of infinitely many variables.

The integration process J, undefined in my earlier theory, is in this theory defined. Its definition turns on the definition of limit which I wish to explain in this note.

By way of example consider on the range $\mathfrak{P}^{III}$ $(p = 1, 2, 3, \ldots, n, \ldots)$ a numerically valued function $\alpha(p)$ with absolutely convergent sum $J\alpha, = \alpha(1) + \alpha(2) + \alpha(3) + \ldots$. $J\alpha$ is the limit in the classical sense, as n increases without bound, of $J_n\alpha, = \alpha(1) + \alpha(2) + \ldots + \alpha(n)$. Here the definitions of $J_n\alpha$ and of limit are of special reference. However, taking (not the first n but) any finite set σ say of n elements p: $p_1 < p_2 < \ldots < p_n$, of the range $\mathfrak{P}^{III}$, we secure definitions of $J_\sigma\alpha, = \alpha(p_1) + \alpha(p_2) + \ldots + \alpha(p_n)$, and of limit which are of general reference.

Indeed, consider at once a general class $\mathfrak{P}$ and a numerically valued (possibly many-valued) function F on the class $\mathfrak{S}$ of all finite sets σ of elements p of the range $\mathfrak{P}$. (In the example cited $F(\sigma) = J_\sigma\alpha$). We say that the number a is the limit as to σ of the function $F(\sigma)$, or that, as to σ, $F(\sigma)$ converges to a, in notation:

$$\lim_{\sigma} F(\sigma) = a, \tag{1}$$

in case for every positive number e there exists a set σ_e (depending on e) of such a nature that for every set σ including σ_e $|F(\sigma) - a| \leqq e$, in symbols:

$$e : \supset : \exists\, \sigma_e \ni \sigma^{\supset \sigma_e} . \supset . |F(\sigma) - a| \leqq e. \tag{2}$$

If for a set σ including σ_e $F(\sigma)$ is many-valued the understanding is that the final inequality holds for every value of $F(\sigma)$. (The notation e denotes a positive number; the notations n, m used below denote positive integers.)

The L_σ of (1) is a single-valued functional operation of the type of a definite integral, in that it reduces every function $F(\sigma)$ of the class of all functions convergent as to σ to a number a. Accordingly, the class $\mathfrak{P}$ being general, this definition of limit (even apart from its use in the theory mentioned) belongs to General Integral Analysis.

In order to obtain definitions of various modes of convergence in case the function F involves a parameter we notice the equivalent forms (3, 4, 5) of the definition.

$$n : \supset : \exists\, \sigma_n \ni \sigma^{\supset \sigma_n} . \supset . |F(\sigma) - a| \leqq 1/n, \tag{3}$$

$$\exists\, \{\sigma_m\} \ni n : \supset : \sigma^{\supset \sigma_n} . \supset . |F(\sigma) - a| \leqq 1/n. \tag{4}$$

viz., there exists a sequence $\{\sigma_m\} : \sigma_1, \sigma_2, \ldots, \sigma_m, \ldots$, of sets σ of such a nature that for every n and set σ including σ_n $|\ F(\sigma) - a\ | \leqq 1/n$.

$$\exists\{\sigma_m\} \cdot \ni n : \supset : \exists m_n \ni \sigma^{\supset \sigma_{m_n}} . \supset . |\ F(\sigma) - a\ | \leqq 1/n, \qquad (5)$$

viz., there exists a sequence $\{\sigma_m\}$ such that for every positive integer n there exists a positive integer m_n (depending on n) such that for every set σ including σ_{m_n} $|\ F(\sigma) - a\ | \leqq 1/n$.

Now let the function F involve a parameter u on a range $\mathfrak{U}$ and suppose that $F(\sigma, u)$ converges as to σ for every u of $\mathfrak{U}$; the limit is a single-valued function, say φ, of u, in notation:

$$\lim_{\sigma} F(\sigma, u) = \varphi(u) \qquad (u), \qquad (6)$$

that is,

$$u. : \supset :. \exists\{\sigma_{um}\} \ni n : \supset : \exists m_{un} \ni \sigma^{\supset \sigma_{um_{un}}} . \supset . |\ F(\sigma, u) - \varphi(u)\ | \leqq 1/n, \qquad (7)$$

viz., for every u of $\mathfrak{U}$ there exists a sequence $\{\sigma_{um}\}$ of sets σ (depending on u) such that for every positive integer n there exists a positive integer m_{un} (depending on u and n) such that for every σ including $\sigma_{um_{un}}$ $|\ F(\sigma, u) - \varphi(u)\ | \leqq 1/n$.

The convergence is semiuniform over the range $\mathfrak{U}$ in case a single sequence $\{\sigma_m\}$ is effective as the sequence $\{\sigma_{un}\}$ for every u of $\mathfrak{U}$, and it is uniform in case moreover for every n a single positive integer m_n is effective as the positive integer m_{un} for every u of $\mathfrak{U}$, that is, the notations:

$$\lim_{\sigma} F(\sigma, u) = \varphi(u) \qquad (u;\ \text{semiunif.}); \qquad (8)$$

$$\lim_{\sigma} F(\sigma, u) = \varphi(u) \qquad (u;\ \text{unif.}), \qquad (9)$$

have the respective meanings:

$$\exists\{\sigma_m\} \ni u. : \supset :. n : \supset : \exists m_{un} \ni \sigma^{\supset \sigma_{m_{un}}} . \supset . |\ F(\sigma, u) - \varphi(u)\ | \leqq 1/n; \qquad (10)$$

$$\exists\{\sigma_m\} \ni n. : \supset :. \exists m_n \ni u : \supset : \sigma^{\supset \sigma_{m_n}} . \supset . |\ F(\sigma, u) - \varphi(u)\ | \leqq 1/n. \qquad (11)$$

If v is a numerically valued single-valued function of u on $\mathfrak{U}$, we define semiuniformity and uniformity of convergence relative to v as scale function over the range $\mathfrak{U}$, in notation: as in (8, 9) with the parentheses replaced by (u; semiunif. $v(u)$), (u; unif. $v(u)$) respectively, by replacing in the definitions (10, 11) the final $1/n$ by $|\ v(u)\ |/n$. Thus, semiuniformity and uniformity are absolute, i.e., relative to the scale function $v(u) = 1$.

The definitions of uniformity (absolute and relative) may be simplified by omitting '$\exists\, m_n\ni$' and replacing 'σ_{m_n}' by 'σ_n'. This form of definition is suggested directly by (4); the more complicated form (5) with its redundant existential feature m_n serves however to suggest the definitions of semiuniformity (absolute and relative), and relative semiuniformity proves to be of importance in the applications.

[1] M. Fréchet, Sur quelques points du calcul fonctionnel, *Palermo, Rend. Circ. mat.*, **22**, 1–74 (1906).

[2] E. H. Moore, Introduction to a Form of General Analysis, 1–150, *The New Haven Mathematical Colloquium*, Yale Univ. Press, 1910. Cf. also, E. H. Moore, On a Form of General Analysis with Application to Linear Differential and Integral Equations, *Atti* IV *Cong. Inter. Mat.* (Roma, 1908), **2**, 98–114 (1909).

[3] E. H. Moore, On the Foundations of the Theory of Linear Integral Equations, *Bull. Amer. Math. Soc.*, Ser. 2, **18**, 334–362, (1912). On the Fundamental Functional Operation of a General Theory of Linear Integral Equations, *Proc. Fifth Inter. Congr. Math.* (Cambridge, Aug., 1912) **1**, 230–255 (1913).

NOTICE OF SCIENTIFIC MEMOIR

The Variations and Ecological Distribution of the Snails of the Genus Io. By CHARLES C. ADAMS, New York State College of Forestry, Syracuse, N. Y. Second Memoir of Volume 12 of the Memoirs of the National Academy of Sciences, Washington, 1915. 1–184 p., 64 pl.

Io is a large gilled snail which lives only in the Tennessee River system. It is extremely variable, shows a remarkable distribution in the streams and in this it appears to be related to the physical history of the drainage. Throughout this Memoir emphasis is placed upon relating the changes of the animals to the changes in the environment. The general natural history of the snails is briefly summarized, the local races are described and the localities from which the collections studied were secured is given in detail. The shells were grouped in convenient classes for descriptive purposes. The diameter of the shell, its degree of globosity, and the degree of development of the spines were determined quantitatively. These qualities are discussed by streams and drainage systems. In the parallel flowing Powell, Clinch and (North Fork) Holston rivers, the shells are smooth or with low spines in the headwaters, and down stream have longer spines. This condition is quite remarkable and no previous detailed investigation has been made of a problem of this character.

The development or evolution of the gross environment is discussed. The author states "It is considered that a knowledge of the development and structure of the environment is as essential a part of the problem as is the development and structure of the animals themselves." An outline history of the Tennessee drainage is given. In the past this family of shells was thought to have originated in the Northwest (Laramie) but the author suggests an alternative hypothesis, that they originated in the southeast.

Large numbers of immature shells were secured in order to study the development of the shell and its sculpture. Certain shells are smooth or without sculpture in the early stages, some remain smooth, others develop nodules or spines either early or late in life, or even develop a sculpture and later lose it. Other shells are spinose from the start and remain so. The loss of sculpture occurs largely in one locality, in the vicinity of which drainage changes have probably occurred in relatively recent geological time.

In the general discussion and summary a brief review is given of inheritance in molluscs, as Mendelian inheritance applies to many of their characters. The results of recent experimental breedings are applied to the interpretation of the variations of these shells. Other topics discussed are agencies and means of dispersal, longitudinal distribution in streams, causes of variation, geographic range and variation of each of the forms of *Io*, the migration of the environment and migrations and relationships of the various forms. The illustrations include figures of the shells, showing the ontogeny of shell sculpture, and samples of the shell population at representative localities. Diagrams and tables illustrate the quantitative relations and there are several maps. The great diversity seen in these shells is probably related to the complex mixtures of many strains which has been made possible by the complex stream history.

REPORT OF THE AUTUMN MEETING

Prepared by the Home Secretary

The Autumn Meeting of the Academy was held in the American Museum of Natural History at New York City, on November 15, 16, 17, 1915.

Forty-eight members were present, as follows: C. G. Abbot, Becker, Boas, Boltwood, Britton, Castle, Cattell, Chittenden, W. B. Clark, F. W. Clarke, J. M. Clarke, Conklin, Coulter, Crew, Cross, Davenport, Day, Donaldson, Hague, Harper, Harrison, Hillebrand, Holmes, Howell, Iddings, Lindgren, Loeb, Lusk, Mall, Mendel, Michelson, Morgan, Morse, A. A. Noyes, H. F. Osborn, T. B. Osborne, Pickering, Ransome, Reid, Richards, Rosa, Schuchert, Theobald Smith, Van Hise, Walcott, Webster, Wheeler, H. S. White.

BUSINESS SESSION.

The Home Secretary announced that since the Annual Meeting in April the Academy had lost by death two members, John Ulric Nef, elected 1904, who died on August 13, 1915, and Frederick Ward Putnam, elected 1885, who died on August 18, 1915; also two foreign associates, Theodor Boveri, elected 1913, who died on October 15, 1915, and Paul Ehrlich, elected 1904, who died on August 20, 1915.

The Home Secretary made also the following announcements: That Mr. Edwin B. Frost had been elected Chairman of the Directors of the Bache Fund and Mr. Arthur Gordon Webster had been elected to succeed Mr. Ira Remsen.

That Mr. Theodore William Richards had been elected to succeed Mr. Ira Remsen as a member of the Directors of the Wolcott Gibbs Fund.

That the ad interim appointments of Acting Chairmen of the Sections of the Academy were as follows:

MATHEMATICS: E. H. MOORE, Acting Chairman.
ASTRONOMY: G. C. COMSTOCK, Acting Chairman.
PHYSICS AND ENGINEERING: R. S. WOODWARD, Acting Chairman.
CHEMISTRY: A. A. NOYES, Acting Chairman.
GEOLOGY AND PALEONTOLOGY: ARNOLD HAGUE, Acting Chairman.
BOTANY: J. M. COULTER, Chairman, 1915–1918.
ZOOLOGY AND ANIMAL MORPHOLOGY: E. G. CONKLIN, Acting Chairman.
PHYSIOLOGY AND PATHOLOGY: R. H. CHITTENDEN, Acting Chairman.
ANTHROPOLOGY AND PSYCHOLOGY: W. H. HOLMES, Acting Chairman.

That the following members were appointed an Auditing Committee to audit the accounts of the Treasurer in accordance with Rule V. 5.

ARTHUR L. DAY WILLIAM H. DALL, Chairman F. W. CLARKE

That the division of the members of the Board of Editors of the Proceedings into three groups, one-third of them retiring annually, in accordance with Rule V. 1, was approved as follows:

Dec. 1, 1916	Dec. 1, 1917	Dec. 1, 1918
W. B. CANNON	E. G. CONKLIN	J. J. ABEL
J. MCK. CATTELL	C. B. DAVENPORT	J. M. CLARKE
J. M. COULTER	E. B. FROST	J. P. IDDINGS
SIMON FLEXNER	W. H. HOLMES	R. A. MILLIKAN
R. G. HARRISON	E. H. MOORE	ALEXANDER SMITH

The Committee on the Henry Draper Fund recommended to the Academy that the following appropriations be made from available income: Four hundred dollars to Doctor Frank Schlesinger, Director of the Allegheny Observatory, to be applied toward the construction of a machine for measuring astronomical photographs; three hundred dollars to Doctor Joel Stebbins, Director of the University of Illinois Observatory, to be used in the further development of the photo-electric photometer and its application to the study of variable stars; two hundred dollars, to be applied toward the expense of publishing the Proceedings of the National Academy.

SCIENTIFIC SESSIONS

A public lecture was given on November 15 by Michael I. Pupin, of Columbia University, on Problems of Aerial Transmission.

Three public scientific sessions were held on November 16 and 17 at which the following papers were presented:

1. EDWIN G. CONKLIN: Nature of cell polarity.
2. W. E. CASTLE: Is selection or mutation the more important agency in evolution?
3. A. E. VERRILL: Inheritance of clubbed feet through five known generations; inheritance of abnormal or defective thumbs through four known generations.

4. CHAS. B. DAVENPORT: Heredity of stature.

5. E. C. MACDOWELL (introduced by Chas. B. Davenport): Parental alcoholism and mental ability—A comparative study of habit formation.

6. JAMES B. MURPHY (introduced by Jacques Loeb): Rôle of the lymphocytes in resistance to cancer.

7. GRAHAM LUSK: The calorimeter as an interpreter of life processes.

8. T. B. OSBORNE and LAFAYETTE B. MENDEL: The resumption of growth after failure to grow.

9. W. H. HOWELL: Ultramicroscopic studies of the fibrin-gel.

10. C. WILLIAM BEEBE (introduced by Henry Fairfield Osborn): Origin of the flight of birds.

11. FRANK M. CHAPMAN (introduced by Henry Fairfield Osborn): Ornithological survey of the Andies and Western Coast of South America.

12. DOUGLAS HOUGHTON CAMPBELL: Treubia.

13. MARSHALL A. HOWE (introduced by N. L. Britton): Fossil calcareous algæ from the Panama Canal Zone with reference to reef-building algae.

14. A. B. STOUT (introduced by N. L. Britton): Sterility in plants and its inheritance.

15. J. N. ROSE (introduced by N. L. Britton): Recent explorations in the cactus deserts of South America.

16. GEORGE H. SHULL (introduced by Chas. B. Davenport): Some factors affecting the inheritance ratios in shepherd's purse.

17. HERBERT M. RICHARDS (introduced by R. A. Harper): The respiratory ratio of cacti in relation to their acidity.

18. R. A. HARPER: Some studies in morphogenesis.

19. HERBERT S. JENNINGS: Can we observe organic evolution in progress?

20. JOHN M. COULTER: Orthogenesis in plants.

21. THEODORE W. RICHARDS: Investigations recently conducted in the Wolcott Gibbs Memorial Laboratory.

22. B. B. BOLTWOOD: The life of radium.

23. ALFRED G. MAYER and ROBERT S. WOODWARD (by title): The biography of Alfred Marshall Mayer.

24. G. C. ABBOT: The solar radiation and its variability.

25. A. G. WEBSTER: Experiments and theory of conical horns; instruments for measurements and sound; an instrument for finding the direction of a fog-signal.

26. EDWARD C. PICKERING: The New Draper Catalogue.

27. HENRY NORRIS RUSSELL (introduced by Edward C. Pickering): On the albedo of the moon and planets.

28. GEORGE F. BECKER: A possible origin for some spiral nebulae.

29. L. A. BAUER (introduced by R. S. Woodward): Concomitant changes in the earth's magnetism and solar radiation.

30. FRED E. WRIGHT and J. C. HOSTETTER (introduced by Arthur L. Day): Experiments on the mean free path of gases; observations on Wood's one-dimensional gas.

31. JAMES KENDALL (introduced by Alexander Smith): The water correction in conductivity determinations.

32. HENRY FAIRFIELD OSBORN: Extremes of adaptation in carnivorous dinosaurs, tyrannosaurus and ornithomimus.

33. C. K. LEITH (introduced by C. R. Van Hise): The influence of certain minerals on the development of schists and gneisses.

34. W. M. DAVIS: Glacial sculpture of the Mission Range, Montana.

35. WALDEMAR LINDGREN: Crystallization of quartz veins.

36. GEORGE P. MERRILL (introduced by Arthur L. Day): The minor constitutents of meteorites.

37. E. W. HILGARD: A peculiar clay from near the City of Mexico.

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Errata

p. 325, l. 23, *for* four hundredth *read* three hundredth.
p. 325, last line, *delete* (instead of 150).
p. 498, l. 2, *for* 2.01 *read* 20.1.